Math F401: Homework 7 Supplement **Due: October 20**

1. Determine (with proof) which of the following series converge.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+3}}{n^2-2n+12}$

c. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3^n}$ *Hint:* $3 = (\sqrt{3})^2$.

2. (Hand in to David)

We define a function $f : [0,1] \to \mathbb{R}$ by

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1/q & \text{if } x = p/q \text{ where } p \text{ and } q \text{ are relatively prime integers (i.e. } \gcd(p,q)=1\text{).} \end{cases}$$

So for example, $f(1) = 1$, $f(1/2) = 1/2$, $f(\pi/4) = 0$, $f(17/29) = 1/29$, and so forth.

a. Make a sketch of the graph of the function f. Clearly you won't be able to do an exact job here, but try to convey the important features of the function.

b. Prove that given $c \in [0,1]$ and $q \in \mathbb{N}$ there exists an $\epsilon > 0$ such that there are no points x of the form p/q such $0 < |x-c| < \epsilon$.

c. For what values of $c \in [0,1]$ does $\lim_{x\to c} f(x)$ exist? Prove your claim.