Math F401: Homework 3 Supplement **Due: September 25**

1. (Hand in to David) An algebraic number is a solution of an equation of the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 = 0$$

where each of the a_i's is an integer. That is, an algebraic number is a root of a polynomial with integer coefficients.

a. Show that $\sqrt{2}$ and $\sqrt{2} + \sqrt{3}$ are algebraic numbers.

b. Prove that the set of algebraic numbers is countable. You may freely use the fact that a polynomial of degree n has no more than n distinct roots.

2. This problem is a variation of a result proved in class. (You may use the class result in proving this homework problem). Let $S \subseteq \mathbb{R}$. Show that $x = \sup S$ if and only if

a. x is an upper bound for S, and

b. for every $n \in \mathbb{N}$, there is $s_n \in S$ such that $x - \frac{1}{n} < s_n \leq x$.