1. (Hand in to David) The following theorem was stated in class without proof. Prove it.

Suppose $f : [a, b] \to \mathbb{R}$ is Riemann integrable and that $\mathcal{P}$ is a partition of $[a, b]$ such that $U(\mathcal{P}, f) - L(\mathcal{P}, f) < \epsilon$. Suppose $\mathcal{P}^*$ is any tagging of $\mathcal{P}$. Then

$$\left| S(\mathcal{P}^*, f) - \int_a^b f \right| < \epsilon.$$