Math F401: Homework 11 **Due: November 29**

1. State the definition of a Riemann integrable function $f : [a,b] \to \mathbb{R}$.

2. Consider the function $f(x) = |x|$ on $[-1,1]$. Let $\epsilon > 0$. Find (with proof) a partition $\mathcal{P}$ such that

$$U(\mathcal{P}, f) - L(\mathcal{P}, f) < \epsilon.$$

3. Suppose $f : [a,b] \to \mathbb{R}$ is continuous and that $f(x) \geq 0$ for every $x \in [a,b]$. Show that if $\int_a^b f = 0$, then $f \equiv 0$. *Hint:* Use the contrapositive and compare with a suitable step function.

4. Bartle & Sherbert 7.2.2

5. Bartle & Sherbert 7.2.10

6. Bartle & Sherbert 7.2.16

7. **(Hand in to David)** Let $f : [a,b] \to \mathbb{R}$ be Riemann integrable. Show that $|f|$ is also Riemann integrable and $\left|\int_a^b f\right| \leq \int_a^b |f|$. *Hint:* Use the characterization that a function is Riemann integrable if and only if for every $\epsilon > 0$ there is a partition $\mathcal{P}$ with $U(\mathcal{P}, f) - L(\mathcal{P}, f) < \epsilon$.

8.

- **a.** Define $f : [a,b] \to \mathbb{R}$ by $f(x) = 0$ except for $x = x_0 \in [a,b]$, where $f(x) = 1$. Show that f is Riemann integrable and that $\int_a^b f = 0$.
- **b.** Let f be a Riemann integrable function on $[a,b]$ and let g be a function on $[a,b]$ that is equal to f everywhere except at finitely many points. Show that g is Riemann integrable and $\int_a^b g = \int_a^b f$.

9. **(Hand in to David)** Find an example of a Riemann integrable function that is discontinuous at infinitely many points.

10. Let $f : [a,b] \to \mathbb{R}$ be Riemann integrable. Show that f^2 is Riemann integrable.