

(. Hand in to David) A subset U of the real numbers is said to be **open** if for every $x \in U$ there is an $\epsilon > 0$ such that the set $(x - \epsilon, x + \epsilon)$ is contained in U . A subset of the real numbers is said to be **closed** if its complement is open.

- a. Show that a closed interval $[a, b]$ is a closed set.
- b. Prove that if $\{x_n\}_{n=1}^{\infty}$ is a sequence where each $x_n \in U$ for some closed set U , and if $\lim_{n \rightarrow \infty} x_n = x$, then $x \in U$.
- c. Prove that if A is a closed and bounded set, and if $f : A \rightarrow \mathbb{R}$ is continuous, then f is bounded.