Recall the following fact:

Theorem 1. *Suppose f is a continuous real valued function defined on an interval containing arbitrarily small numbers (e.g. $(0, \pi)$) and that f satisfies $f(x + y) = f(x) + f(y)$ whenever both sides are well defined. Then there is a constant c such that $f(x) = cx$.*

1. A **biangle** or **lune** on a sphere is a region enclosed by two distinct great circles. Let $A(\theta)$ denote the area of a lune with interior angle θ. Make a diagram with your copy of the sphere to show that A is an additive function. (i.e. that $A(\theta_1 + \theta_2) = A(\theta_1) + A(\theta_2)$).

2. Find a formula for $A(\theta)$ in terms of θ and the area S of the sphere.

3. Form a triangle on your copy of a sphere using great circles for sides. How many copies of that triangle appear on the sphere?

4. Use your diagram from part 3 to determine the number of lunes that overlap your triangle.

5. Find a formula relating the area of the sphere, the area of your triangle, and the area of the lunes on your sphere.

6. Find a formula relating the area of a triangle on a sphere and its angle surplus (i.e. $\theta_1 + \theta_2 + \theta_3 - 180$).