Math F651: Homework 8 **Due: March 25**

1.

Show that a topological space X is Hausdorff if and only if every net in X converges to at most one pont.

2.

Show that if X and Y are topological spaces, then a net $\langle (x_\alpha, y_\alpha) \rangle_{\alpha\in A}$ in $X \times Y$ converges to (x, y) if and only if $x_\alpha \to x$ and $y_\alpha \to y$.

3.

Show that a net $\langle x_\alpha \rangle_{\alpha\in A}$ in a topological space X converges to x if and only if every subnet of $\langle x_\alpha \rangle_{\alpha\in A}$ has a sub-subnet converging to x. (This is a net version of a property of sequences: a sequence $\{x_i\}$ converges to x if and only if every subsequence has a sub-subsequence converging to x.

4.

In a previous homework, you gave a proof of exercise 26.8. Give a new solution to this problem using nets. Tell me about which method of proof you prefer.

5.

a. Suppose G is a topological group. Suppose that $\langle g_\alpha \rangle_{\alpha\in A}$ and $\langle h_\alpha \rangle_{\alpha\in A}$ are nets in G such that $g_\alpha \to g$ and $g_\alpha h_\alpha \to f$. Show that $h_\alpha \to g^{-1}f$.

b. (Problem 11 on page 188): Show that if A and B are subsets of a topological group G such that A is compact and B is closed, then $A \cdot B$ is closed. If you want, also try your hand at proving this result net-free.