These problems from Munkres:

- Page 145 #4 (no rigor here, just answers)

As well as:

1.

Let Z be the subspace $(\mathbb{R} \times 0) \cup (0 \times \mathbb{R})$ of $\mathbb{R}^2$. Define $g : \mathbb{R}^2 \to Z$ by the equations

$$\begin{aligned} g(x, y) &= (x, 0) \qquad \text{if } x \neq 0 \\ g(0, y) &= (0, y). \end{aligned}$$

a. Is g a quotient map?

b. Show that in the quotient topology induced by g, the space Z is not Hausdorff.

2.

Let X be the line with two zeros. That is, X is the quotient space of $\{0, 1\} \times \mathbb{R}$ given by the equivalence relation $(0, x) \sim (1, x)$ if $x \neq 0$. We showed in class that X is not Hausdorff. Give a careful proof that X is locally Euclidean. Also show that X is second countable.

3.

Suppose a topological group G acts continuously on a topological space X. Show that the quotient map $\pi : X \to X/G$ is an open map.

4.

Suppose G is a topological group and H is a subgroup of G. Show that if H is a normal subgroup of G, then G/H is a topological group. You will find your result from problem 3 to be useful.