These problems from Munkres:

- Page 92 #9
- Page 101 #14
- Page 118 #7

As well as:

**1.**

An $n$-dimensional manifold with boundary is a second countable Hausdorff space $M$ such that every point $x \in M$ has an open neighborhood $U$ homeomorphic to an open subset of the upper half space $\mathbb{R}^{n,+} = \{x \in \mathbb{R}^n : x_n \geq 0\}$. Every such homeomorphsim is called a chart. The terminology "manifold with boundary" is historical and mildly misleading: every manifold is a manifold with boundary, but a manifold with boundary typically isn't a manfold! The boundary of $M$, denoted by $\partial M$, is the set of points $x \in M$ such that there is a chart taking $x$ to a point of $\partial\mathbb{R}^{n,+}$. This notion of boundary is not the same as the topological notion of boundary we gave in class. Since the topological boundary of $M$ is the empty set (and therefore not particularly interesting), there is usuallly no confusion in using the symbol $\partial M$ to mean something different for manifolds with boundary.

Anyway, enough my rambling. Here's the problem: show that $\partial M$ is an $(n-1)$ dimensional manifold.

**2.**

Let $X$ be a first countable space.

a. Show that if $A \subset X$ is any set, then $x \in \overline{A}$ if and only if there is a sequence $\{x_i\}_{i=1}^{\infty}$ such that $x_i \to x$.

b. Show that if $Y$ is some other topological space, and if $f : X \to Y$, then $f$ is continuous if and only if it takes convergent sequences to convergent sequences.