Math F651: Homework 2 **January 28, 2005**

These problems from Munkres:

- Page 84 #8b
- Page 92 #7
- Page 111 #7a

As well as:

1.

The point of the following exercise is to show that it is easy to be a continuous map into a topological space with a coarse topology, or to be a continuous map from a space with a fine topology (and conversely harder to go the other way). Let X and Y be a topological spaces and let X' and Y' be the same point-sets but with finer topologies. Prove the following.

1 If $f : X \to Y'$ is continuous, then $X \to Y$ is continuous.

2 If $f : X \to Y$ is continuous, then $X' \to Y$ is continuous.

3 Give examples where the converses of 1) and 2) are false.

4 Let X_d be the set X with the discrete topology. Every function $f : X_d \to Y$ is continuous.

5 Let Y_i be the set Y with the indiscrete topology. Every function $f : X \to Y_i$ is continuous.

6 If Z is a Hausdorff space, the only continuous maps from $Y_i \to Z$ are the constants.

7 What are the continous maps are from $\mathbb{R}$ to X_d? From $\mathbb{R} - \{0\}$ to X_d?

2.

Let I be the interval $(0, 1)$. Let X be the set $I \times I$ with the topology inherited from $\mathbb{R}^2$ with the order topology, and let Y be the same set with the usual metric topology. Determine if either of the identity maps $X \to Y$ or $Y \to X$ are continuous.

3.

If $f : X \to Y$ is a map between topological spaces, we say that f is **open** if $f(U)$ is open for every open set in X. Suppose $f : X \to Y$ is an open continous map.

1 Show that f is a homeomorphism if and only if f is bijective.

2 Show that if f is surjective, and if $\mathcal{B}$ is a basis for X, then the collection $\{f(B) : B \in \mathcal{B}\}$ is a basis for Y.

3 Find a map from $\mathbb{R}^2$ to $\mathbb{R}^2$ that is open but not continuous.