These problems from Munkres:

- Page 84 #8b
- Page 92 #7
- Page 111 #7a

As well as:

1. The point of the following exercise is to show that it is easy to be a continuous map into a topological space with a coarse topology, or to be a continuous map from a space with a fine topology (and conversely harder to go the other way). Let $X$ and $Y$ be a topological spaces and let $X'$ and $Y'$ be the same point-sets but with finer topologies. Prove the following.

   1. If $f : X \to Y'$ is continuous, then $X \to Y$ is continuous.
   2. If $f : X \to Y$ is continuous, then $X' \to Y$ is continuous.
   3. Give examples where the converses of 1) and 2) are false.
   4. Let $X_d$ be the set $X$ with the discrete topology. Every function $f : X_d \to Y$ is continuous.
   5. Let $Y_i$ be the set $Y$ with the indiscrete topology. Every function $f : X \to Y_i$ is continuous.
   6. If $Z$ is a Hausdorff space, the only continuous maps from $Y_i \to Z$ are the constants.
   7. What are the continuous maps from $\mathbb{R}$ to $X_d$? From $\mathbb{R} - \{0\}$ to $X_d$?

2. Let $I$ be the interval $(0, 1)$. Let $X$ be the set $I \times I$ with the topology inherited from $\mathbb{R}^2$ with the order topology, and let $Y$ be the same set with the usual metric topology. Determine if either of the identity maps $X \to Y$ or $Y \to X$ are continuous.

3. If $f : X \to Y$ is a map between topological spaces, we say that $f$ is open if $f(U)$ is open for every open set in $X$. Suppose $f : X \to Y$ is an open continuous map.

   1. Show that $f$ is a homeomorphism if and only if $f$ is bijective.
   2. Show that if $f$ is surjective, and if $\mathcal{B}$ is a basis for $X$, then the collection $\{f(B) : B \in \mathcal{B}\}$ is a basis for $Y$.
   3. Find a map from $\mathbb{R}^2$ to $\mathbb{R}^2$ that is open but not continuous.