1: Munkres 53.4.

2: Munkres 59.3.

3: The Brower fixed point theorem in dimension 2. Let \( B^2 \) be the closed unit ball in \( \mathbb{R}^2 \). Show that for any continuous function \( F : B^2 \to B^2 \), there is a point \( x \in B^2 \) such that \( F(x) = x \).

\[ \text{Hint: Suppose there is no fixed point and consider the map } G : S^1 \to S^1 \text{ given by } G(x) = \frac{x - F(x)}{|x - F(x)|}. \]
Show that \( G \) is homotopic to the identity map on \( S^1 \) and also that \( G \) is null homotopic to derive a contradiction. Note: Your book has a different, longer proof of this result. You might find Lemma 55.3 useful in your proof.

4. a. Let \( X \) be a topological space and let \( H : I \times I \to X \) be continuous. Define paths

\[
\begin{align*}
f(t) &= H(0, t) \\
g(t) &= H(t, 1) \\
h(t) &= H(t, 0) \\
k(t) &= H(1, t)
\end{align*}
\]
show that \( f \cdot g \sim h \cdot k \).

b. Show that the fundamental group of any topological group \( G \) at any base point is abelian.

\[ \text{Hint: Given loops } \alpha \text{ and } \beta \text{ in } G \text{ based at the identity, consider the map } H : I \times I \to G \text{ defined by } H(s, t) = \alpha(s)\beta(t). \]

5. Let \( d : \pi_1(S^1, 1) \to \mathbb{Z} \) be the isomorphism we defined in class given by

\[
d([f]) = \tilde{f}(1)
\]
where \( \tilde{f} \) is the lift of \( f \) such that \( \tilde{f}(0) = 0 \). For any map \( F : S^1 \to S^1 \) we define the degree of \( F \) via

\[
\deg(F) = d(\hat{F}_*([\epsilon]))
\]
where \( \epsilon(t) = e^{2\pi it} \) and where \( \hat{F}(z) = F(z)/F(1) \).

a. Show that two maps \( F \) and \( G \) from \( S^1 \) to \( S^1 \) are homotopic if and only if they have the same degree.

b. Show that \( \deg(F \circ G) = \deg(F) \deg(G) \) for maps \( F, G \) from \( S^1 \) to \( S^1 \).

6. The Klein bottle is \( \mathbb{R}^2/\mathbb{Z}^2 \) where \( \mathbb{Z}^2 \) acts on \( \mathbb{R}^2 \) by \( (n, m) \cdot (x, y) = (x + n, (-1)^n y + m) \).
Find a two sheeted covering of the Klein bottle by the torus.