Math F651: Homework 12 **Due: April 21**

1: Munkres 58.5.

2: Munkres 58.6.

3. Let X be a path connected space and let p and q be distinct points in X. Let h be a path from p to q. We defined a map in class $\Phi_h : \pi_1(X,p) \to \pi_1(X,q)$ by

$$\Phi_h([f]) = [h^{-1}] \cdot [f] \cdot [h]$$

and we showed that this map is a group isomorphism. In general the isomorphism Φ_g depends on the choice of $[g]$. Find (and prove) an algebraic necessary and sufficient condition on $\pi_1(X,p)$ such that Φ_g does not depend on $[g]$. (If you go hunting in Munkres, you can find out what the condition is, but try not to!)

4. Show that if A is a retract of a Hausdorff space, then A is closed.

5. The Möbius band is $\mathbb{R}^2/\mathbb{Z}$ where $\mathbb{Z}$ acts on $\mathbb{R}^2$ by $n \cdot (x,y) = (x+n, (-1)^n y)$. Give a careful proof that the Möbius band is homotopy equivalent to the circle. This question is as much a question about carefully working with quotient spaces as it is about homotopy. The characteristic property of the quotient topology should appear in your proof.

6. Show that the torus with a point removed is homotopy equivalent to a figure of eight. A proof by pictures is fine for this problem. *Hint:* The torus is a quotient of a square.