Munkres 32.1. Show that every locally compact Hausdorff space is regular. Find a counterexample to show that a locally compact Hausdorff space need not be normal.

Munkres 32.2. Show that for a metric space $X$ the following are equivalent:

1. $X$ is second countable.
2. $X$ has a countable dense subset (i.e. $X$ is separable).
3. Every open cover of $X$ has a countable subcover (i.e. $X$ is Lindelöf).

Munkres 33.4. Let $X$ be a normal space. Show that if $A \subset X$, then there is a continuous function $f : X \to \mathbb{R}$ such that $f^{-1}(0) = A$ if and only if $A$ a closed $G_\delta$ set. (A $G_\delta$ set is a countable intersection of open sets.) Show that if such a function exists, it can be taken so that $f(X) \subset [0, 1]$.

*Hint:* See Theorem 21.6 of your text and the definition of uniform convergence which precedes it.

Munkres 33.5. Let $X$ be a normal space, and let $A$ and $B$ be subsets of $X$ Show that there is a continuous function $f : X \to [0, 1]$ such that $f^{-1}(A) = 0$ and $f^{-1}(B) = 1$ if and only if $A$ and $B$ are disjoint closed $G_\delta$ sets. (Try to find as short a solution as possible here!)

Munkres 34.3. Let $X$ be a compact Hausdorff space. Show that $X$ is metrizable if and only if it is second countable.