Math F651: Homework 1 Supplemental Problem **January 21, 2005**

Prove the following proposition

Proposition 1. *Let d_1 and d_2 be two metrics on a set X. The following condiditions are equivalent.*

1. *For every sequence $\{p_i\}_{i=1}^{\infty}$, if $p_i \underset{d_2}{\longrightarrow} p$ then $p_i \underset{d_1}{\longrightarrow} p$.*
2. *For every function $f : X \to \mathbb{R}$, if f is continuous with respect to d_1 then f is continuous with respect to d_2.*
3. *For every set U, if U is open with respect to d_1 then U is open with respecet to d_2.*

Hint: You might want to show 1) $\iff$ 2) and 1) $\iff$ 3).

It will be helpful to recall the following definitions and results from metric space theory. Let (X, d) be a metric space and let V be a subset of X. We say that p is a **limit point** of V if there is a sequence $\{p_i\}_{i=1}^{\infty}$ such that $p_i \in V$ and such that $p_i \to p$. We say that a set V is **closed** if and only if V contains its limit points. Finally, it is a result that a set V is closed if and only if its complement is open.