

These problems from do Carmo:

- 3-2: #3, #4

As well as:

1. Compute the second fundamental form of the helicoid in the coordinates defined by the chart $\sigma(u, v) = (v \cos(u), v \sin(u), u)$.
2. Compute the normal and geodesic curvature of the lines of latitude of the unit sphere.
3. Suppose S is an orientable surface and that $\sigma : \mathbb{R}^2 \rightarrow S$ is a chart such that the second fundamental form in these coordinates vanishes. Prove that $\sigma(\mathbb{R}^2)$ is contained in a plane. (This is a version for surfaces of the fact that if a curve has vanishing curvature, then its trace is contained in a line).

Hint: Use the fact that

$$\frac{d}{du}N(\sigma(u, v)) = DN_{\sigma(u, v)}(\sigma_u(u, v))$$

and so forth (along with the definition of the second fundamental form) to show that the normal vector is constant.

See me for more hints if need be!