Math F412: Homework 8 **Due: March 28, 2005**

These problems from do Carmo:

- 2-5 #12

As well as:

1. Consider the patch

$$\sigma(u,v) = (\cosh(v)\cos(u), \cosh(v)\sin(u), v)$$

for $0 < u < 2\pi$ that defines the catenoid with a line removed, and the patch

$$\hat{\sigma}(u,v) = (v\cos(u), v\sin(u), u)$$

for $0 < u < 2\pi$ that defines the portion of the helicoid between $z = 0$ and $z = 2\pi$.

a Sketch both surfaces.

b Show that the map taking $\sigma(u,v)$ to $\hat{\sigma}(u, \sinh(v))$ is an isometry.

2. Let S be the upper half cylinder defined by $S = \{(x,y,z) : x^2 + y^2 = 1, z > 0\}$. Let $\hat{S}$ be the punctured unit ball:

$$\hat{S} = \{(x,y,z) : x^2 + y^2 < 1, z = 0, (x,y,z) \neq (0,0,0)\}.$$

Find a conformal map taking S to $\hat{S}$. Feel free to ask for hints if need be!

3. Let S be the one-sheeted hyperboloid defined by $x^2 + y^2 = 1 + z^2$. For each $z \in \mathbb{R}$, let

$$\alpha_z(t) = ((1+z^2)\cos(t), (1+z^2)\sin(t), z),$$

which is a curve in S. Compute the normal curvature and geodesic curvature of α_z.