Math F412: Homework 3 Supplemental Problems **January 28, 2005**

1.

Recall that $O(3)$ is the set of 3x3 orthogonal matrices and that $SO(3)$ is the set of orthogonal matrices with determinent equal to 1.

a. Show that if v and w are vectors in $\mathbb{R}^3$ and A is any matrix, then $\langle Av, w\rangle = \left\langle v, A^T w\right\rangle$.

b. Show that if u, v, and w are vectors in $\mathbb{R}^3$, and if A is any matrix, then $\det(Au, Av, Aw) = \det(A)\det(u, v, w)$.

c. Show that if $U \in SO(3)$ and u and v are vectors in $\mathbb{R}^3$, then $(Uu)\wedge(Uv) = U(u\wedge v)$. *Hint:* Show that $U(u \wedge v)$ satsifies

$$\langle U(u \wedge v), w\rangle = \det(Uu, Uv, w)$$

for any vector w and use the definition of the vector product on page 12 of do Carmo.

2.

Suppose $\alpha(s)$ is a smooth curve with curvature $k(s) > 0$ and torsion $\tau(s)$ (parameterized by unit speed, for simplicity). If $U \in O(3)$ and $\det(U) = -1$, compute the curvature and torsion of the curve $\hat{\alpha}(t) = U\alpha(t)$.