Math F412: Homework 10   **Due: April 8, 2005**

These problems from do Carmo:

- 3-2: 5, 7

- 3-3: 5abc

Also:

1.   Compute the principle curvatures of a surface in terms of K and H.

2.   An **asymptotic curve** in a surface is a curve that has 0 normal curvature everywhere. Show that if α is an asymptotic curvature, then $K(\alpha(t)) < 0$ for every t in the domain of α.

3.   The **tractrix** is the curve given by

$$\alpha(v) = (\sin(v), \cos(v) + \log(\tan(v/2))).$$

You saw this curve before in the first homework (problem 1-3.4).

a.   Sketch the trace of this curve for $v \in (0, \pi/2)$.

b.   Let S be the surface of revolution of this curve (with the domain $(0, \pi/2)$. Sketch this surface and compute the Gaussian and mean curvatures of S. (Be careful about using Example 4 of section 3-3: the curve α does not have unit speed!)

c.   Notice that the Gaussian curvature of S is a negative constant. For this reason we call S the pseudosphere. But any surface with constant negative Gaussian curvature is quite different from the sphere. Name two properties that the sphere has that a surface with constant negative Gaussian curvature cannot have (besides having (constant) positive Gaussian curvature!).