1. **Vocabulary:**
   - Amortization of loans
   - Amortization schedule

2. **Loan as an annuity:** A loan that is to be paid back in multiple payments can be thought of as an annuity being bought by the bank.

Suppose a bank loans $1500 and charges interest at APR of 12% and that the loan plus interest is to be paid back by equal payments of amount $R$ at the end of the month for three months.

The three equal payments are an annuity to pay off the amount of the loan plus interest.

To find the equal payment amount, $R$, use the Present Value of an annuity equation and solve for $R$

\[ A = Ra_{\frac{n}{r}} \quad \text{where} \quad a_{\frac{n}{r}} = \frac{1 - (1 + r)^{-n}}{r} \]

\[ R = \frac{A}{a_{\frac{n}{r}}} \]

Here the PV of the annuity is the original is the original loan amount of $1500.

So \[ R = \frac{1500}{a_{\frac{3}{0.1}}} \]

The bank considers each payment to be split in two parts: to pay the interest for the period and the rest to repay the original loan amount a.k.a. principal. This is “killing” off the loan which is where the word AMORTIZING comes from.

Each payment reduces the principal, thus the interest portion decreases each period. The details are given in an **amortization schedule:**

<table>
<thead>
<tr>
<th>Period</th>
<th>Principal at beginning of period</th>
<th>Interest for period</th>
<th>Payment (end)</th>
<th>Principal repaid at end of period</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1500</td>
<td>$15.00</td>
<td>$510.03</td>
<td>495.03</td>
<td>1020.07</td>
</tr>
<tr>
<td>2</td>
<td>1004.97</td>
<td>10.05</td>
<td>510.03</td>
<td>499.98</td>
<td>510.04</td>
</tr>
<tr>
<td>3</td>
<td>504.99</td>
<td>5.05</td>
<td>510.04</td>
<td>504.99</td>
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</tr>
<tr>
<td>Total</td>
<td></td>
<td>30.10</td>
<td>1530.10</td>
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</table>

Note: The last payment is adjusted for rounding.
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To get the **principal at the beginning of each period**, you can think if it as the present value of an annuity with fewer payments. Observe the pattern to determine the number of payments:

- For the second period, there is one less payments.
- For the third period there are two less payments.
- For the kth period there is k−1 less payments,

So number of payments is \( n - (k - 1) = n - k + 1 \)

Gives Formula: 

\[
Ra_{n-k+1|r}
\]

To get the **interest in kth payment**: 

\[
rRa_{n-k+1|r}
\]

To get the **principal contained in kth period** is the payment minus the interest part

\[
R - rRa_{n-k+1|r} = R(1 - a_{n-k+1|r})
\]

**Total interest paid**: 

\[
nR - A = nR - Ra_{n|r} = R(n - a_{n|r})
\]

3. A person amortizes a loan of $170,000 for a new home by obtaining a 20-year mortgage at the rate of 7.5% compounded monthly. Find a) the monthly payment, b) the total interest charges and c) the principal remaining after 5 years.