# Hash Tables

CS 311 Data Structures and Algorithms
Lecture Slides
Monday, November 18, 2024

Glenn G. Chappell
Department of Computer Science
University of Alaska Fairbanks
ggchappell@alaska.edu

Some material contributed by Chris Hartman

# Unit Overview
# Tables & Priority Queues

Topics

- ✓ Introduction to Tables
- ✓ Priority Queues
- ✓ Binary Heap Algorithms
- ✓ Heaps & Priority Queues in the C++ STL
- ✓ 2-3 Trees
- ✓ Other self-balancing search trees
- Hash Tables
- Prefix Trees
- Tables in the C++ STL & Elsewhere

# Review

# Where Are We? — The Big Challenge

Our problem for most of the rest of the semester:

- Store: A collection of data items, all of the same type.
- Operations:
  - Access items [single item: retrieve/find, all items: traverse].
  - Add new item [insert].
  - Eliminate existing item [delete].
- Time & space efficiency are desirable.

Note the three primary single-item operations: **retrieve**, **insert**, **delete**. We will see these over & over again.

A solution to this problem is a **container**.

In a **generic container**, client code can specify the value type.

A **Table** allows for arbitrary key-based look-up.

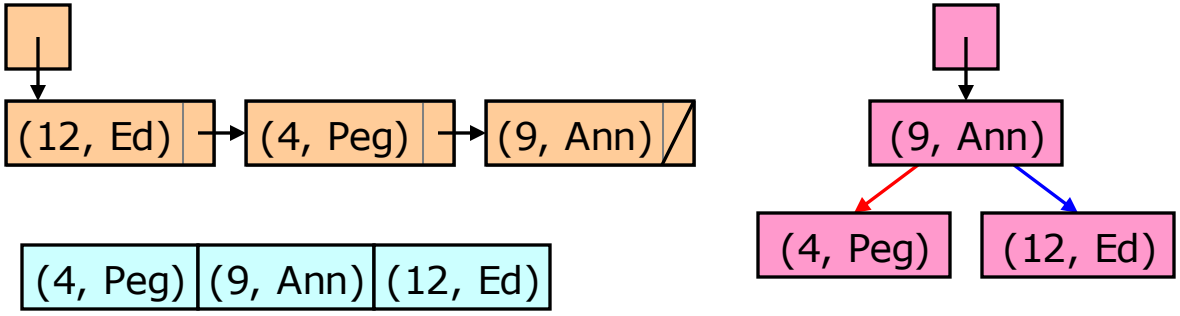
Three single-item operations: retrieve, insert, delete by key.

A Table implementation typically holds **key-value pairs**.

Table

| Key | Value |
|---|---|
| 12 | Ed |
| 4 | Peg |
| 9 | Ann |

Inefficient Implementations




Three ideas for improving efficiency:

1. Restricted Table → Priority Queues
2. Keep a tree balanced → Self-balancing search trees
3. Magic functions → Hash Tables

# Unit Overview
# Tables & Priority Queues

Topics

- ✓ Introduction to Tables ← Several lousy implementations
- ✓ Priority Queues
- ✓ Binary Heap Algorithms
- ✓ Heaps & Priority Queues in the C++ STL

  Idea #1: Restricted Table
- ✓ 2-3 Trees
- ✓ Other self-balancing search trees

  Idea #2: Keep a tree balanced
- Hash Tables

  Idea #3: Magic functions
- Prefix Trees ← A special-purpose implementation: "the Radix Sort of Table implementations"
- Tables in the C++ STL & Elsewhere

# Overview of Advanced Table Implementations

We cover the following advanced Table implementations.

- Self-balancing search trees
  - To make things easier, allow more children (?):
    - ✓ **2-3 Tree**
      - Up to 3 children
    - ✓ **2-3-4 Tree**
      - Up to 4 children
    - ✓ **Red-Black Tree**
      - Binary Tree representation of a 2-3-4 Tree
  - Or back up and try for a strongly balanced Binary Search Tree again:
    - ✓ **AVL Tree**
- Alternatively, forget about trees entirely:
  - **Hash Table**
- Finally, "the Radix Sort of Table implementations":
  - **Prefix Tree**

Idea #2:
Keep a tree balanced

Later, we cover other self-balancing search trees:
B-Trees, B+ Trees.

Idea #3:
Magic functions

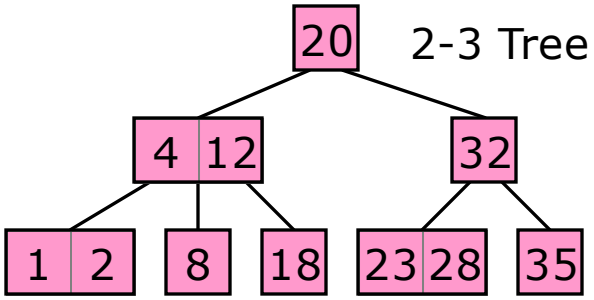
# Review
# 2-3 Trees, Other Self-Balancing Search Trees [1/5]

We looked at four kinds of **self-balancing search trees**:
2-3 Trees, 2-3-4 Trees, Red-Black Trees, and AVL Trees.

All of these have:

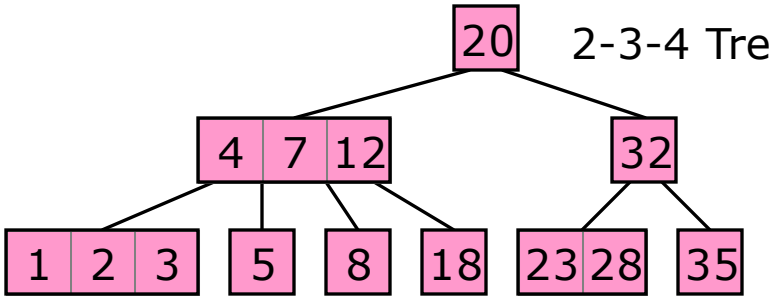
- Θ(log *n*) retrieve, insert, & delete.
- Θ(*n*) traverse (sorted).

**2-3 Tree**: 2-nodes & 3-nodes.
Every node has max number of children or no children.
All leaves at the same level.



2-3 Tree

**2-3-4 Tree**: exactly like a 2-3 Tree, except that 4-nodes are also allowed.



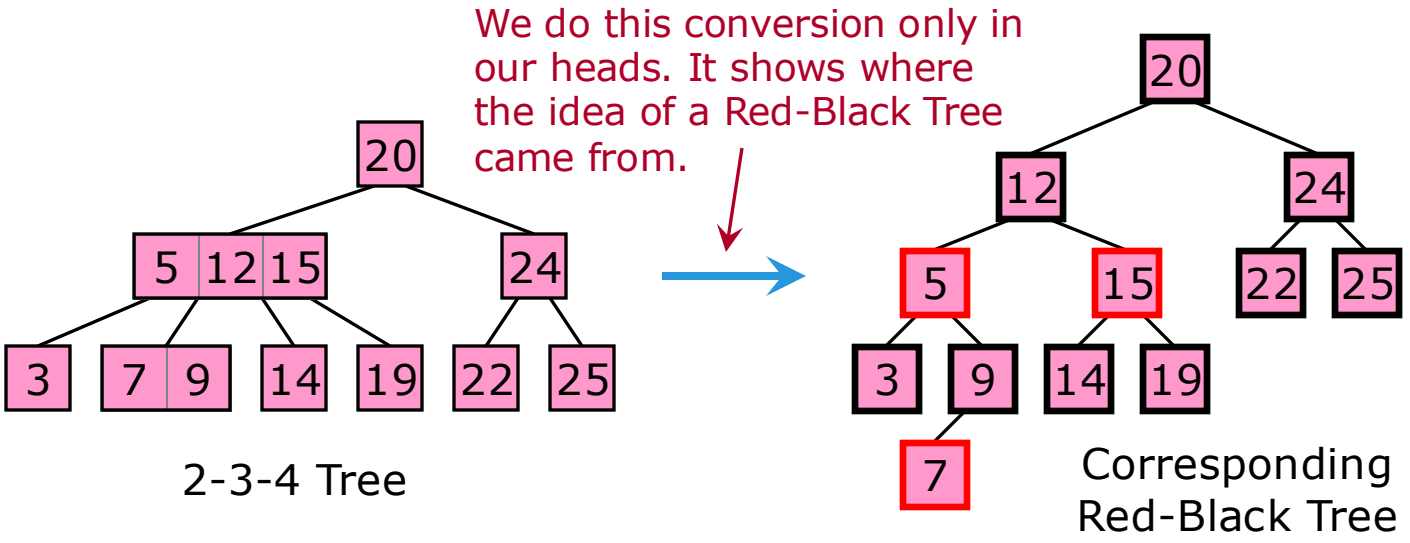
2-3-4 Tree

# 2-3 Trees, Other Self-Balancing Search Trees [2/5]

**Red-Black Tree**: Binary Search Tree representation of 2-3-4 Tree.

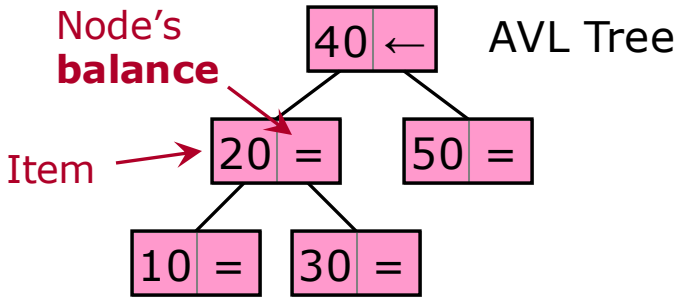
- Each node in a Red-Black Tree is either **red** or **black**.
  - Each **black node** corresponds to a 2-3-4 Tree node.
  - **Red nodes** are extras needed since each node holds only one item.

A Red-Black Tree may not be strongly balanced. However, each path from the root to a leaf hits the same number of **black nodes**, and no more than one **red node** for each **black node**. As a result, Red-Black Trees have logarithmic height.



# 2-3 Trees, Other Self-Balancing Search Trees [3/5]

**AVL Tree**: strongly balanced Binary Search Tree in which each node holds its **balance**: left-high, right-high, or even.




This was the first kind of self-balancing search tree to be developed.

# 2-3 Trees, Other Self-Balancing Search Trees [4/5]

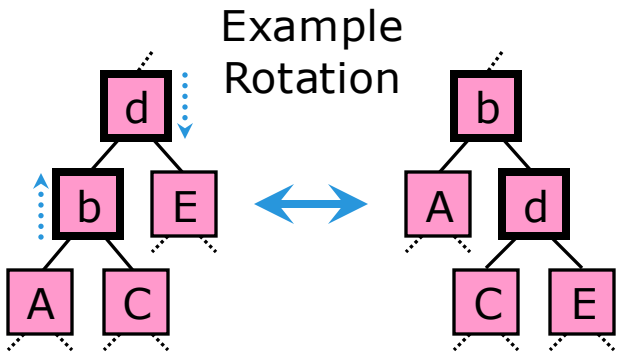
## Summary of Ideas Behind the Algorithms

### Retrieve & (Sorted) Traverse

- For Red-Black Trees & AVL Trees, use the Binary Search Tree algorithms (traverse = inorder traverse).
- For 2-3 Trees & 2-3-4 Trees, use straightforward generalizations of Binary Search Tree algorithms.

### Insert & Delete

- We covered the 2-3 Tree insert & delete algorithms in detail.
- The 2-3-4 Tree algorithms are similar, generally requiring fewer operations (there are typically fewer nodes).




- Algorithms for Red-Black Trees and AVL Trees use **rotations**.

# Review
# 2-3 Trees, Other Self-Balancing Search Trees [5/5]

Generally, the Red-Black Tree is agreed to have the best *overall* performance, for in-memory datasets with many insert & delete operations, when worst-case performance is important.

A Red-Black Tree, or some variant, is the usual implementation for `std::map`, `std::set`, and similar STL containers.

AVL Trees are used less often, but they do have their niche.

2-3 Trees and 2-3-4 Trees are fine data structures. However, they are basically never used, since, in all use cases, some other data structure is a better choice.

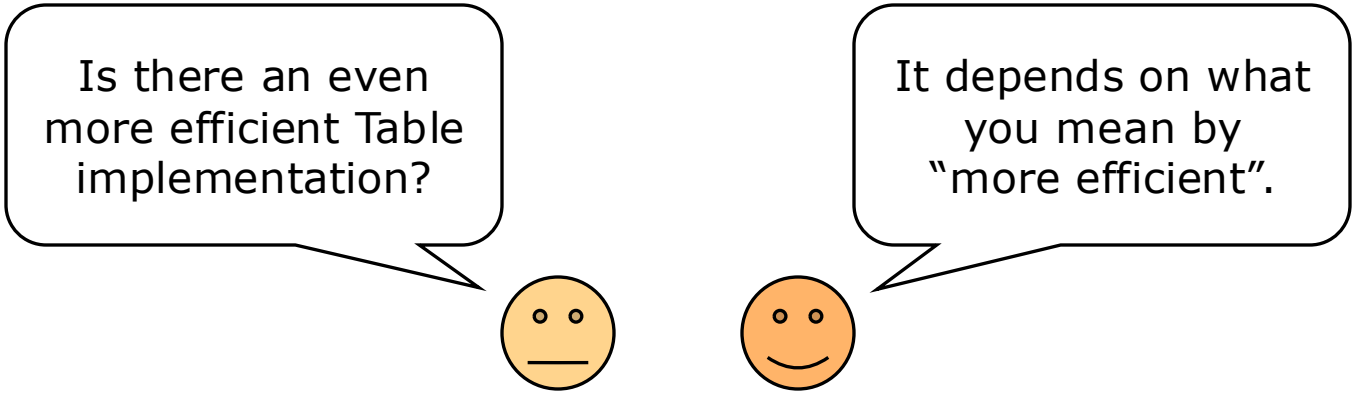
We covered 2-3 Trees and 2-3-4 Trees in order to give an idea of where Red-Black Trees* came from and what they are like, without having to cover all the details.

*Also B-Trees & B+ Trees, when we get to them.

# Hash Tables

# Hash Tables
## Background [1/4]

Self-balancing search trees allow the primary single-item Table operations—retrieve, insert, and delete by key—to be Θ(log *n*).

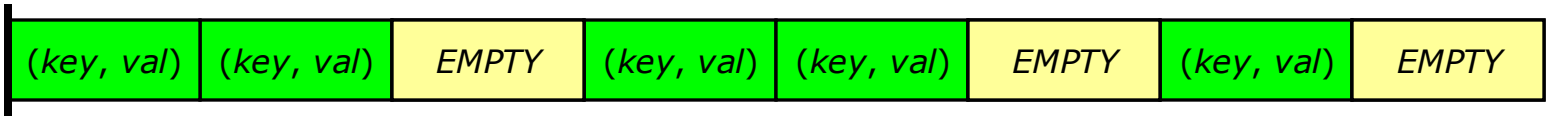



Recall idea #3 for designing an efficient structure to hold an associative dataset: *magic functions*.

Let's see what it leads to …

# Hash Tables
# Background [2/4]

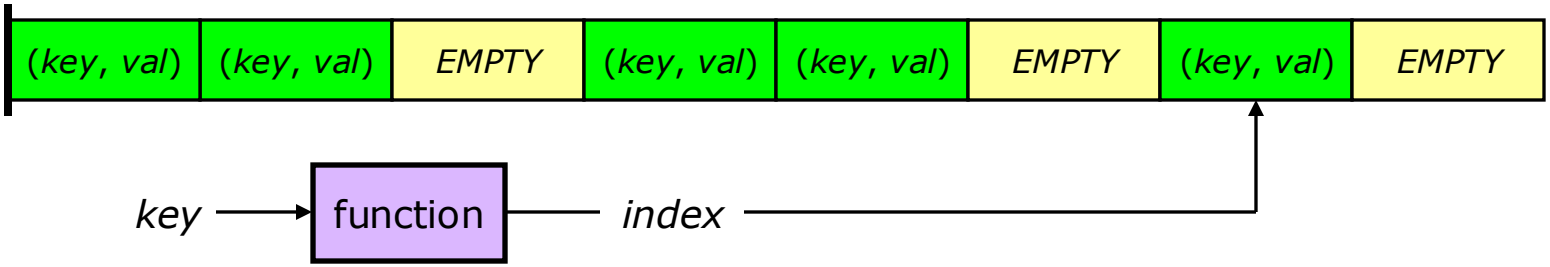
Consider a Table stored as an unsorted array of key-value pairs.




- *Delete by index* can be very fast, if we allow gaps in the data.
  - To delete an item at a given index, just mark it *EMPTY*. Constant time.
- But to do *Table delete*—by key—we must first search by key.
- *Insert by key* would appear to be fast, if we allow duplicate keys.
  - Constant time if no reallocation; generally, amortized constant time.
- But if our dataset does not allow for duplicate keys, then insert requires search by key, to find any existing duplicate key.
- And of course retrieve (by key) requires doing a search by key.
- Unfortunately, search by key is **slow**. We would use Sequential Search: $\Theta(n)$—**or worse**, due to the *EMPTY* spots.

So speeding up search by key might make *everything* fast.

# Hash Tables
## Background [3/4]



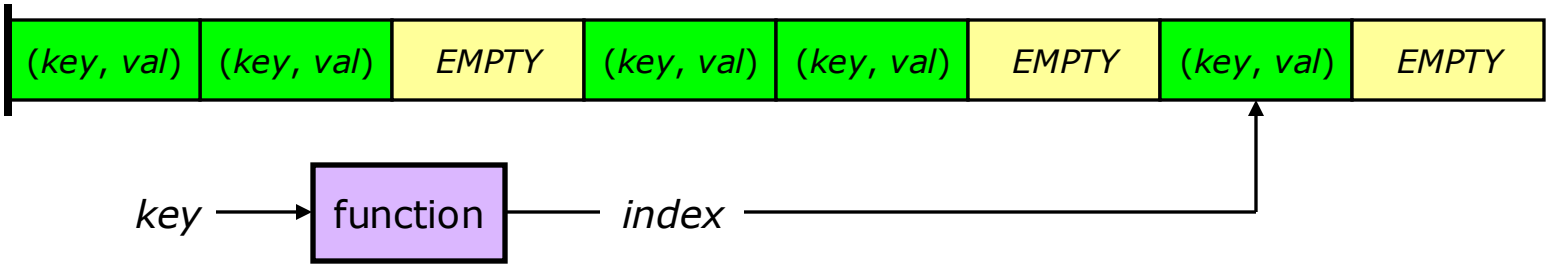

What if we had a magic function that, given a key, returned the index at which the key is stored?

Then all three primary Table operations—retrieve, insert, and delete by key—would be (amortized?) constant time!

Alas, such a function is generally impractical. In common use-cases, it is often actually *impossible*. (What if there are more possible keys than array locations?)

But we can often find a function that is *almost* what we want.

# Hash Tables
# Background [4/4]




So, how do we find a magic *almost*-perfect-key-finder function?

[*Insert high-quality dramatic production here*]

Conclusion: we do not need magic, only *consistency*. For a particular key, the function must always output the same index. The reason the function "knows" where to find an item, is that we asked it where to put the item, when it was inserted.

Data structures based on this idea are called *Hash Tables*.

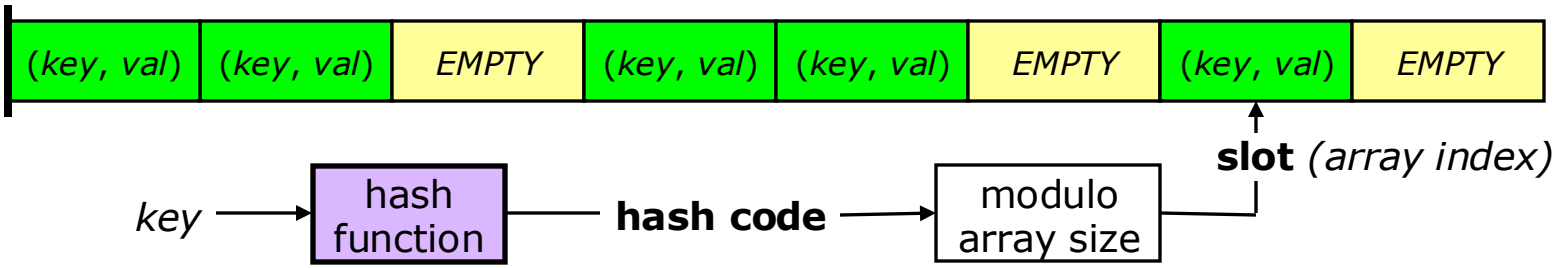
# Hash Tables
# Definitions [1/2]

A **hash function** is a function that "wants to be" our perfect key finder. It takes a key and returns a number: the **hash code**.

- When we pass a key to a hash function, we are **hashing** the key.
- Typically, the function *messes up* its input, so that the output bears little resemblance to the given key. Thus: *hash*.

A **Hash Table** is a data structure in which each item is stored in a location based on a hash code.

- The structure is *something like* an array of key-value pairs (or just keys). Each array index is called a **slot**.
- The hash code may be (much) larger than the size of the array. The slot is typically computed as `hash_code % array_size`.



# Hash Tables
## Definitions [2/2]

The big problem with Hash Tables: the number of possible keys is usually larger than the number of slots. So there *will* be distinct keys that would get the same slot if they were inserted.

- Key sets are often very large. Consider 20-character strings with printable ASCII characters. There are ~$3.6 \times 10^{39}$ such strings.

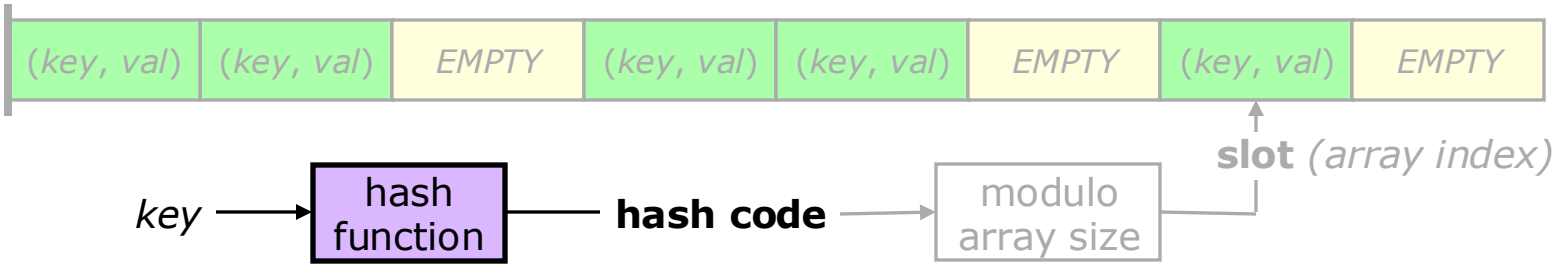
A **collision** happens when an item gets a slot that already holds an item. Dealing with this is called **collision resolution**.

Four issues when putting together a Hash Table:

- What function is used as the hash function?
- How is collision resolution done?
- What if the dataset outgrows the array?
- How efficient is the resulting data structure?

We look at each of these in turn.

# Hash Tables
## Hash Functions — Properties [1/2]



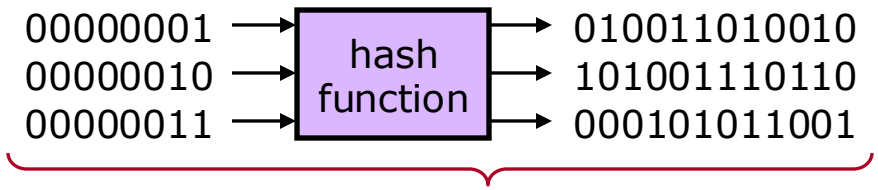

A hash function *must*:

- Take a key and return a nonnegative integer (**hash code**).
- Be **deterministic**: output depends only on input.
  A particular key always gives the same hash code.
- Return the same hash code for equal (==) keys.

Consistency requirement mentioned previously

A *good* hash function:

- Is fast.
- Spreads its results evenly over the possible output values.
- Turns patterns in its input into unpatterned output.



# Hash Functions — Properties [2/2]

The hash function is typically separate from the Hash Table implementation.

- It may be specified by the client code, when the key type is specified.
- It is possible to design a good hash function without knowing a great deal about the Hash Table implementation.

However, the Hash Table implementation will place *some* requirements on the hash function:

- How it is called.
- What it returns.
  - These days, a hash code is typically a 32-bit or 64-bit unsigned integer.
- How exactly the client code specifies a custom hash function.
  - Template/ctor parameters, specially named member functions, etc.
  - Usually, a **hash function** and an **equality comparison** are both given.

# Hash Functions — In Practice [1/4]

A Hash Table implementation typically comes with hash functions for common key types.

- The C++ Standard Library has hash functions for `int`, `bool`, `char`, `double`, `std::string`, `std::shared_ptr`, and other types.

But not *all* possible key types!

**There is no way to generate hash functions automatically for all possible key types.**

So if you provide your own type for the keys, then you also provide a hash function and an equality comparison.

Puzzle

1. Come up with an idea for generating hash functions automatically.
2. Figure out why your idea will not work for some key types.

This issue does not apply to client-provided types used as *associated values*. Only the key needs to be hashed.

## Hash Functions — In Practice [2/4]

Hash functions for standard types are typically accessible by applications. So, for example, if you need to hash a type that is a wrapper around an `int`, then you can easily write your own hash function that calls the standard hash function for `int`.

In all cases, while you might write a hash function yourself, you probably should not design the technique it uses. Designing a good general-purpose hash function requires some expertise and a lot of time. Designing a hash function that is better than those already known is a *huge* challenge.

There are a number of top-notch hash functions to be found on the net. If you need a custom hash function, then do a little research, grab an implementation, and use it!

# Hash Tables
## Hash Functions — In Practice [3/4]

Here is a hash function using a variant of *MurmurHash* [Austin Appleby 2008]. Keys & hashcodes are 32-bit unsigned integers.

```
uint32_t hash(uint32_t key)                 // Based on MurmurHash3
{
    const uint32_t salt = 0xdeadbeefU;  // Value chosen by GGC
    uint32_t h = key * 0xcc9e2d51U;
    h = (h << 15) | (h >> 17);
    h *= 0x1b873593U;
    h ^= salt;
    h = (h << 13) | (h >> 19);
    h = (h * 5) + 0xe6546b64U;
    h ^= 4 ^ (h >> 16);
    h *= 0x85ebca6bU;
    h ^= h >> 13;
    h *= 0xc2b2ae35U;
    return h ^ (h >> 16);
}
```

*See* `hash_function.cpp` *for a program demonstrating this function.*

## Hash Functions — In Practice [4/4]

However, the true measure of a good hash function is the resulting application performance *in actual practice*.

For example, you *may* find that the performance of your application is significantly improved if you switch from the hash function on the previous slide to the following.

```
uint32_t hash(uint32_t key)
{
    return key;
}
```

And if that really is the case—as verified by thorough testing—then you may use this hash function without embarrassment.

## Collision Resolution — Overview [1/2]

**Collision**: when an item gets a slot that already holds an item.

Generally:

- For a well designed hash function, collisions are relatively rare for average data.
- But we typically cannot *guarantee* that there will be few collisions.
- In fact, it is possible that *every* item we insert will be assigned the same index in the array. (Ick!)

Regardless, in a Hash Table that allows arbitrary insertions, we must have a way to deal with collisions.

## Collision Resolution — Overview [2/2]

Collision-resolution methods come in two categories.

**Open Hashing**

- In each slot is a data structure that can store *multiple data items*.
- Each such data structure is called a **bucket**.

**Closed Hashing**

- In each slot there is at most *one data item*.
- If we get a collision, then we look for another slot.

**How collisions are resolved is the primary design decision involved in a Hash Table implementation.**

In **open hashing**:

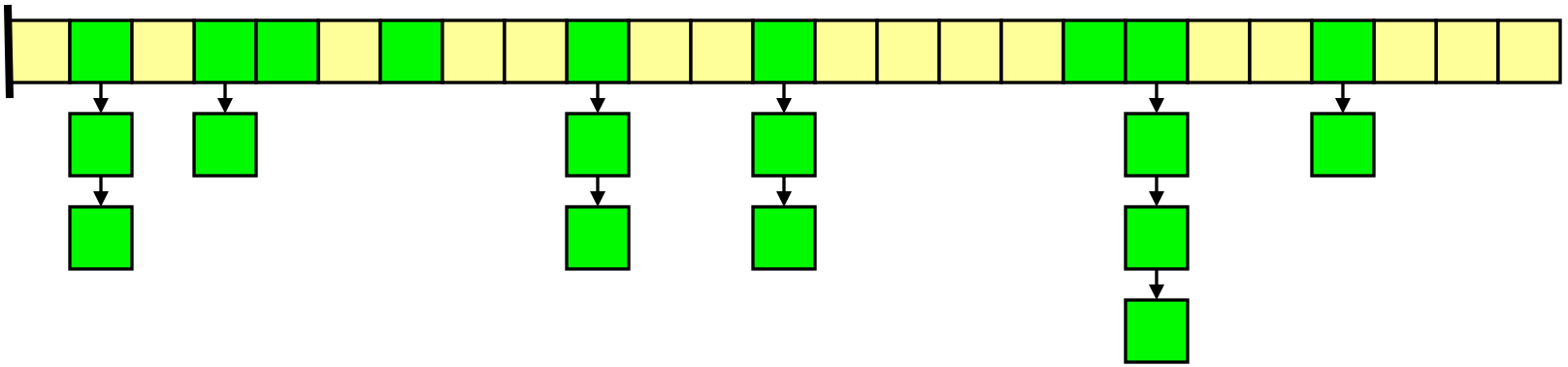
- In each slot is a data structure that can store *multiple data items*.
- Each such data structure is called a **bucket**.
- A search always stays within a single bucket.

If there is a collision:

- Insert a new item in the proper bucket.

Buckets are virtually always Singly Linked Lists.

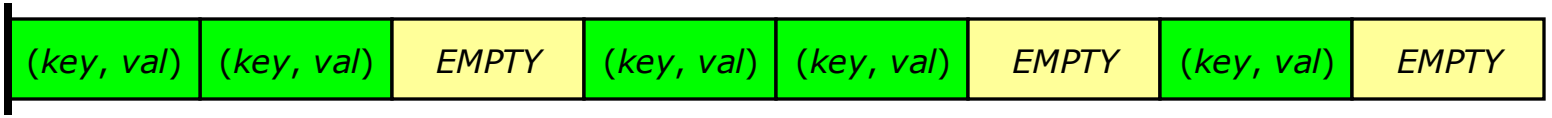
To search for a key: do a Sequential Search in its bucket.



# Collision Resolution — Closed Hashing [1/4]

In **closed hashing**:

- In each slot there is at most *one data item*.
- So the Hash Table is basically an array of data items. However …
- Each slot can be marked as *EMPTY*.




If there is a collision:

- We are inserting a new item, but the slot we want to store it in is already used by an item with a different key.
- So we find a different slot to store the new item in. We keep looking until an empty slot is found.

# Collision Resolution — Closed Hashing [2/4]

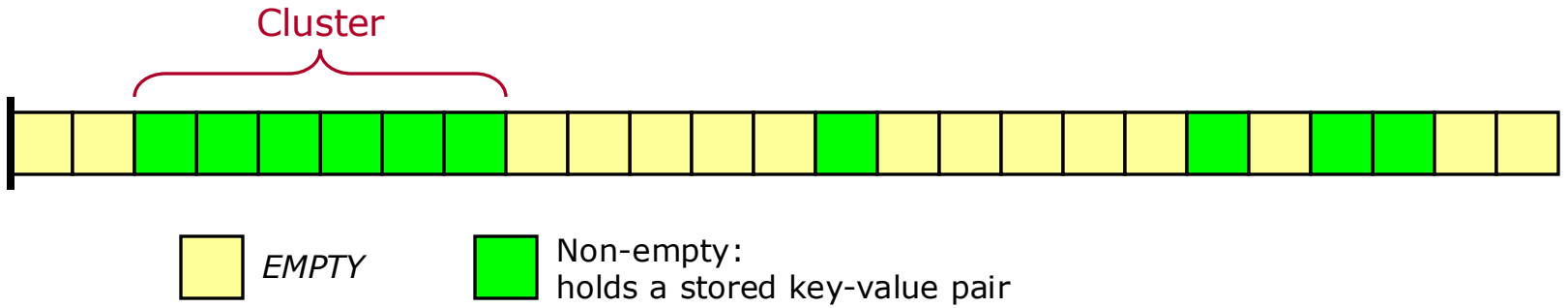
Looking for a given key, or a slot to store a new key, is a **search**.

When doing a search, we check a sequence of slots.

- The first is the slot computed from the hash code.
- Each time we check a slot, we are doing a **probe**.
- The sequence of slots to check is called the **probe sequence**.

The simplest probe sequence is the one in which we look at slot *t*, then *t*+1, *t*+2, *t*+3, etc., wrapping around when we reach the end of the array. This is **linear probing**.

Linear probing tends to form **clusters**, which slow down searches.



## Collision Resolution — Closed Hashing [3/4]

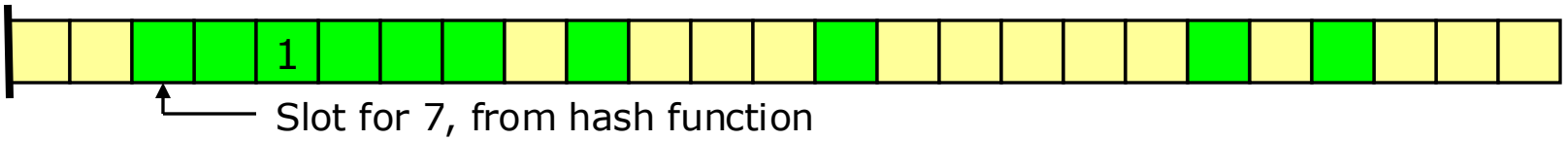
To avoid clusters, we can use **quadratic probing**, which has the probe sequence $t$, $t+1^2$, $t+2^2$, $t+3^2$, etc.

Some techniques involve using a secondary hash function when a collision occurs.

- These generally go under the name of **double hashing**.
- For example, do a variation of linear probing with a step size other than 1. The step size is given by the second hash function.
- Or, just use the second hash function to give a second slot, after which one of the simpler probe sequences is used.

# Collision Resolution — Closed Hashing [4/4]

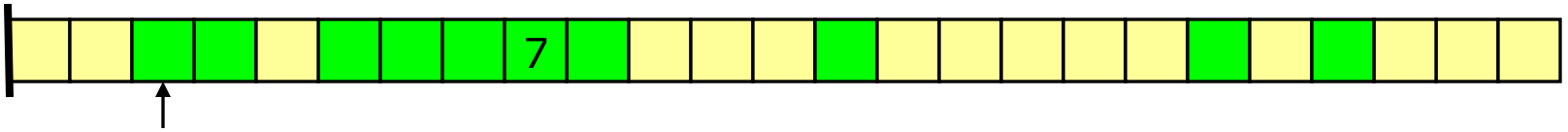
Trickiness in closed hashing: how to be sure a key is *not* present?




In above Hash Table, retrieve 7 (not present). Use linear probing.

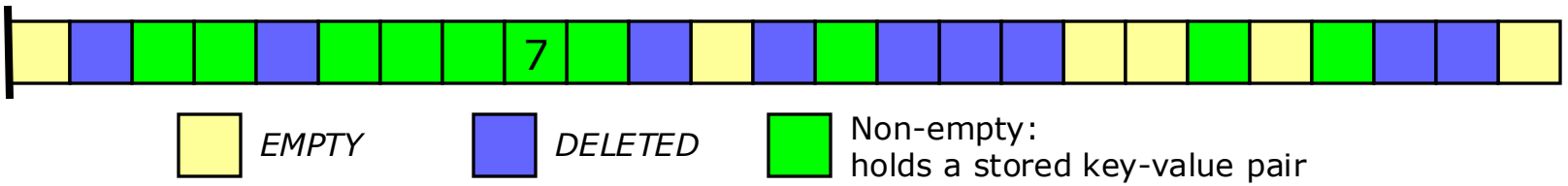
- Begin at arrow, look until we find an *EMPTY*. Return NOT FOUND.

Now insert 7 and then delete 1.




Again, retrieve 7. The above strategy gives NOT FOUND. ☹

Solution. Allow *DELETED* marks. Stop only on key found or *EMPTY*.



All Hash Table implementations that allow insertion will suffer poor performance when the data structure gets too full.

Q. What do we do about this?

A. When the number of items gets too high, we remake the Hash Table, doing a reallocate-and-copy to a larger array—as we did with resizable smart arrays. This is called **rehashing**.

Rehashing is time-consuming. We need to traverse the entire Hash Table, calling the hash function for every key present.

This is one of the downsides of Hash Tables.

Two Questions

1. How do we decide when to do rehashing?
2. How does periodic rehashing affect Hash Table performance?

We look at these after discussing basic Hash Table efficiency.

# Hash Tables
# TO BE CONTINUED …

*Hash Tables* will be continued next time.