Thoughts on Assignment 3
Recursion vs. Iteration

CS 311 Data Structures and Algorithms

Lecture Slides

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Thoughts on Assignment 3

Thoughts on Assignment 3 [1/2]

In Assignment 3, you will be writing multiple functions and function templates relating to recently covered topics:

- Iterators.
- Linked Lists.
- Exceptions.
- Recursion.

Your functions are to be in files `da3.hpp` and `da3.cpp`. You will also need the `llnode.hpp` header. As usual, there is a test program, `da3_test.cpp`, using the *doctest* header: `doctest.h`.

Do not modify `llnode.hpp`.

Thoughts on Assignment 3 [2/2]

To get you started, I have written "skeleton" forms of the files `da3.hpp` and `da3.cpp`; these are in the class Git repository.

These files already compile with the test program. But they fail numerous tests. Your job is to get them working.

Unit Overview
Recursion & Searching

Topics

- ✓ Arrays & Linked Lists
- ✓ Introduction to recursion
- ✓ Search algorithms I
- Recursion vs. iteration
- Search algorithms II
- Eliminating recursion
- Search in the C++ STL
- Recursive backtracking

Review

Review
Search Algorithms I — Binary Search

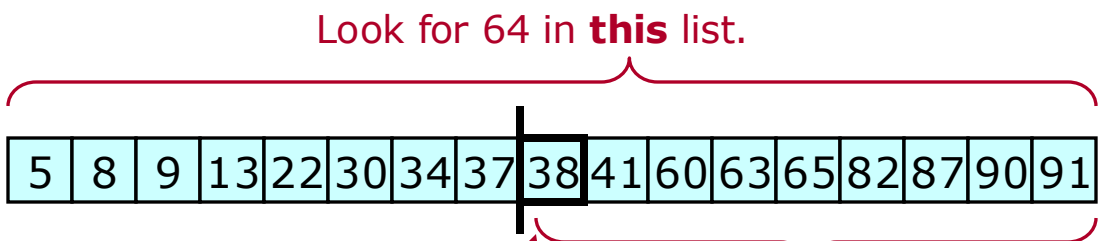
The **Binary Search** algorithm finds a given **key** in a **sorted list**.

- Here, *key* = thing to search for. Often there is associated data.
- In computing, **sorted** means in (some specified) order.

Procedure

- Pick an item in the middle of the list: the **pivot**.
- Compare the given key with the pivot.
- Using this, narrow search to top or bottom half of list. Recurse.

Example. Use Binary Search to search for 64 in the following list.



In practice, a key could be just about anything that can be sorted.

See `binsearch1.cpp`.

Search Algorithms I — Better Binary Search

Equality vs. Equivalence—may not be the same when objects being compared are not numbers.

- **Equality**: `a == b`.
- **Equivalence**: `!(a < b) && !(b < a)`.

Using equivalence instead of equality in Binary Search:

- Maintains consistency: always compare with `operator<`.
- Allows use with value types that do not have `operator==`.

See `binsearch2.cpp`.

Using Operators Random-access iterators only	**Using STL Function Templates** Works with all forward iterators Still fast with random-access
`iter += n`	`std::advance(iter, n)`
`iter + n`	`std::next(iter, n)`
`iter2 - iter1`	`std::distance(iter1, iter2)`

Recursion vs. Iteration

Recursion vs. Iteration
Definitions

There are two ways for code to repeatedly perform an operation an arbitrary number of times.

- **Iteration.** Using one or more loops.
  Code that performs iteration is said to be **iterative**.
- **Recursion.** When a function calls itself.
  Code that performs recursion is said to be **recursive**.

Now we look closer at these two.

Along the way, we will compute Fibonacci numbers using several different methods.

Recursion vs. Iteration
Fibonacci Again — Faster

We wrote a function that, given *n*, returns Fibonacci number *n*. For *n* > 40, our function is extremely slow.

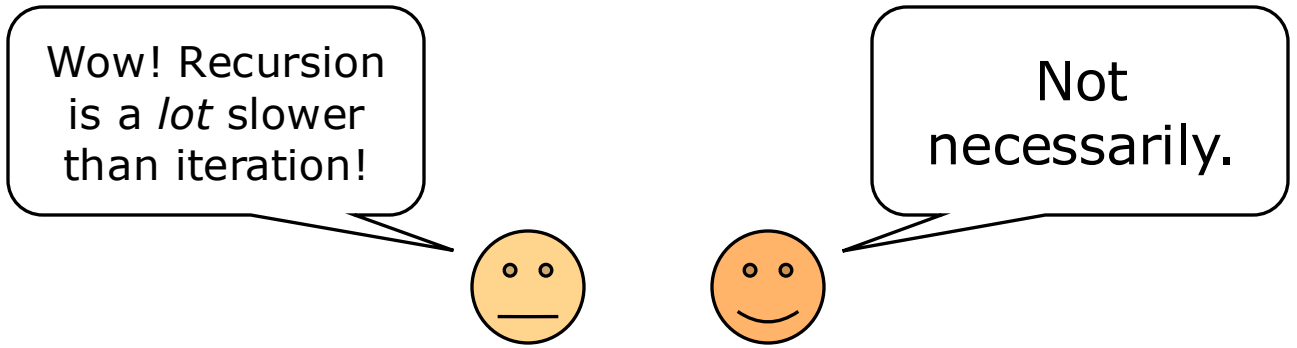
See `fibo_first.cpp`.

What can we do about this?

TO DO

- Rewrite the Fibonacci computation in a fast iterative form.

Done. See `fibo_iterate.cpp`.



TO DO

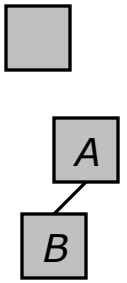
- Figure out how to do a fast recursive Fibonacci computation. Write it.

Done. See `fibo_recurse.cpp`.

Recursion vs. Iteration
Fibonacci Again — Using Trees

Use a **tree** to represent function calls some algorithm makes.

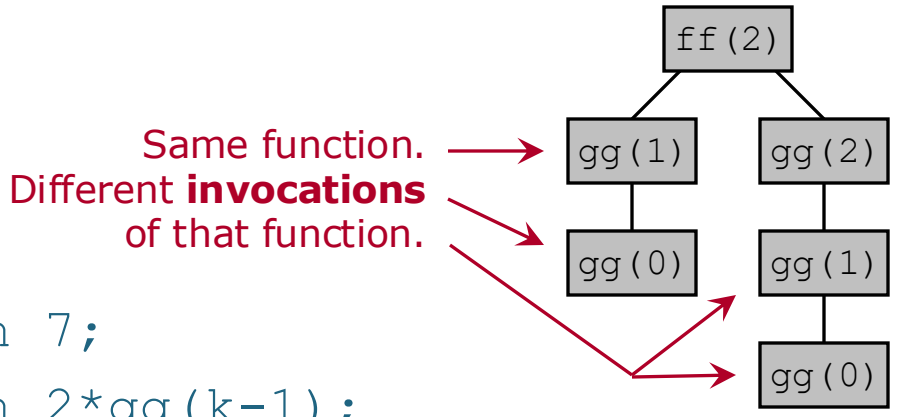
- A box represents making a call to a function.
- A line from an *A* box down to a *B* box represents this call to function *A* making a call to function *B*.




```
int ff(int n)
{
    return gg(n-1) + gg(n);
}


int gg(int k)
{
    if (k == 0) return 7;
    else        return 2*gg(k-1);
}
```

Tree representing calls made by doing `ff(2)`



(Yes, our trees are upside-down.)

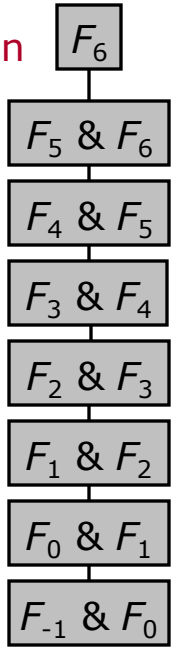
Recursion vs. Iteration
Fibonacci Again — Comments [1/3]

Choice of algorithm can make a *huge* difference in performance.

Computing F_6

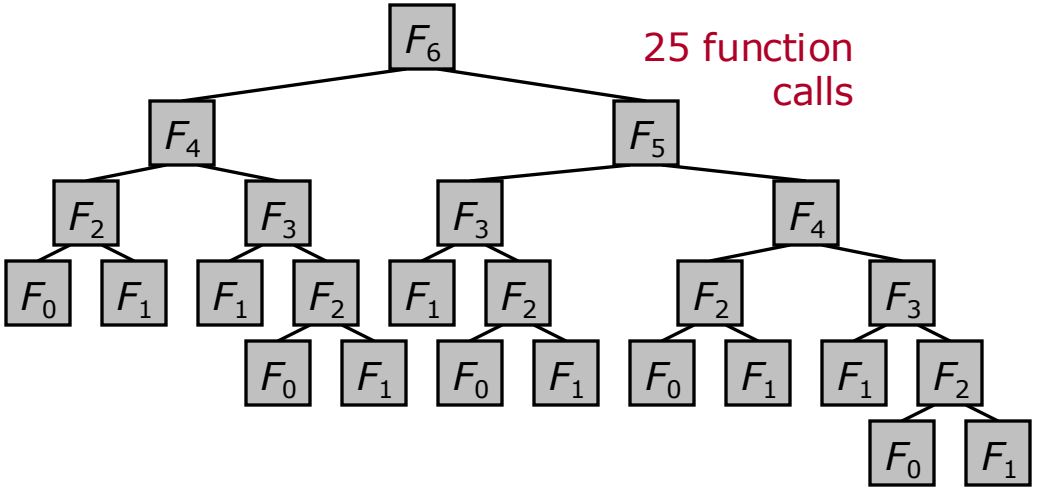
`fibo_recurse.cpp`

8 function calls



`fibo_first.cpp`

25 function calls



Fibonacci No.	`fibo_recurse.cpp`	`fibo_first.cpp`
F_7	9 calls	41 calls
F_{10}	12 calls	177 calls
F_{20}	22 calls	21,891 calls
F_{40}	42 calls	331,160,281 calls

Recursion vs. Iteration
Fibonacci Again — Comments [2/3]

A `struct` can be used to return two values at once. Templates `std::pair` (`<utility>`) and `std::tuple` (`<tuple>`) can be helpful.

The 2017 C++ Standard introduced **structured bindings**, making this more convenient.

```
pair<bignum, bignum> fibo_recurse(int n);

auto [a, b] = fibo_recurse(k);
```

Now `a` and `b` are variables of type `bignum`.

`a` is `fibo(k-1)`.

`b` is `fibo(k)`.

Recursion vs. Iteration
Fibonacci Again — Comments [3/3]

Some algorithms have natural implementations in both **recursive** and **iterative** form.

Sometimes we have a **workhorse** function that does most of the processing, and a **wrapper** function with a convenient interface.

- Often the wrapper just calls the workhorse for us.
- This is common when we use recursion, since recursion can place restrictions on how a function is called.

We have seen this idea in another context. Recall `toString` and `operator<<` from Assignment 1.

```
cout << p.toString();
```

If we had not written our own `operator<<`, then we could still do this.

```
cout << p;
```

With our `operator<<`, we can do this. So `operator<<` is really just a convenient wrapper around `toString`.

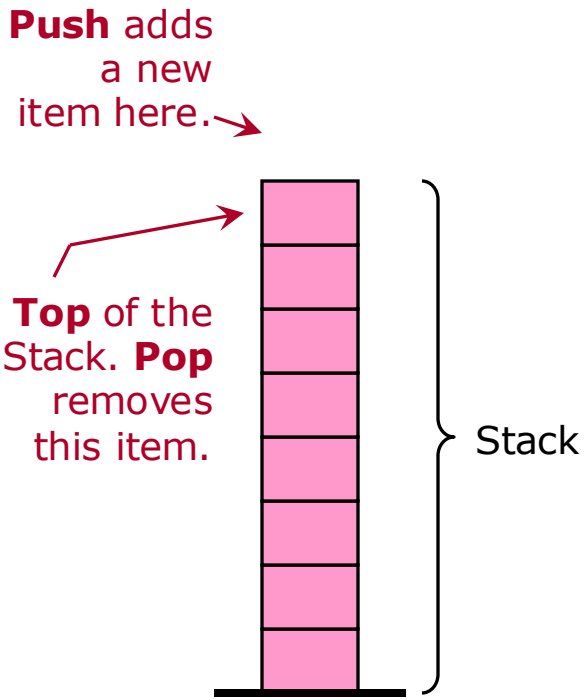
Recursion vs. Iteration
Function-Call Internals [1/4]

To fully grasp the issues involved in recursion vs. iteration, it helps to understand how function calls work in a running program.

A running program makes use of a structure called the **call stack**. (There are other names, all involving the word "stack".)

A Stack is a kind of container. We look at Stacks in detail later in the semester. For now:

- Think of a stack of plates. We can place a plate on top or pull a plate off the top. We only deal with the **top** of the Stack.
- Taking off the top item is a **pop**.
- Adding a new item on top is a **push**.



Recursion vs. Iteration
Function-Call Internals [2/4]

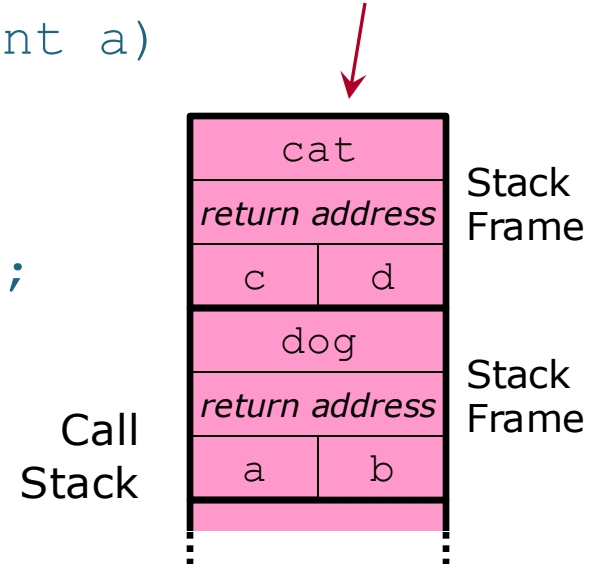
The items on the call stack are **stack frames**. Each stack frame corresponds to an *invocation* of a function.

- A function's stack frame holds:
  - Its automatic variables, including parameters.
  - Its **return address**: where to go back to when it returns.
- When a function is called, a stack frame for that function is pushed.
- When the function exits, its stack frame is popped.

```
void cat(Foo c)
{
    int d;
    llama();
    …
}

void dog(int a)
{
    Foo b;
    cat(b);
}
```

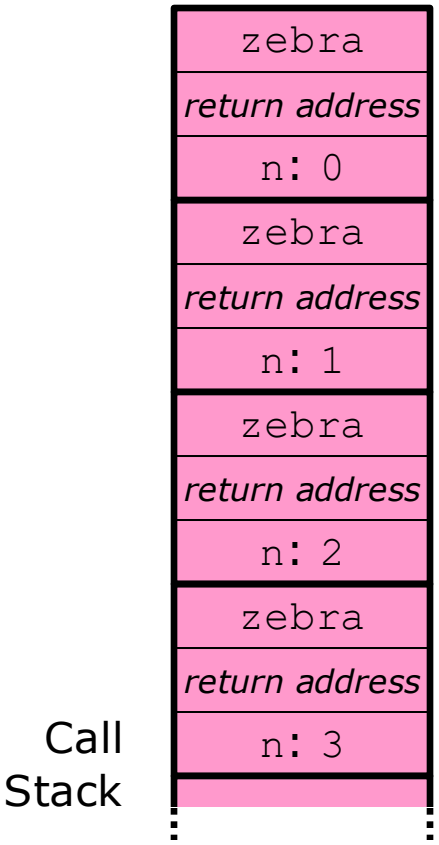
When `cat` is called by `dog`, at **this** point in the code, the call stack will look like **this**.



Recursion vs. Iteration
Function-Call Internals [3/4]

When a function calls itself recursively, there will be multiple stack frames on the call stack corresponding to the *same* function—but *different invocations* of that function.

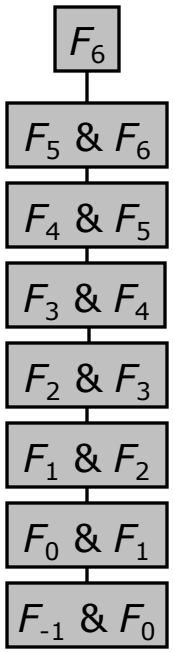
```
void zebra(int n)
{
    if (n == 0)
    {
        cout << n << endl;
        return;
    }
    cout << n << " ";
    zebra(n-1);
}
```



Recursion vs. Iteration
Function-Call Internals [4/4]

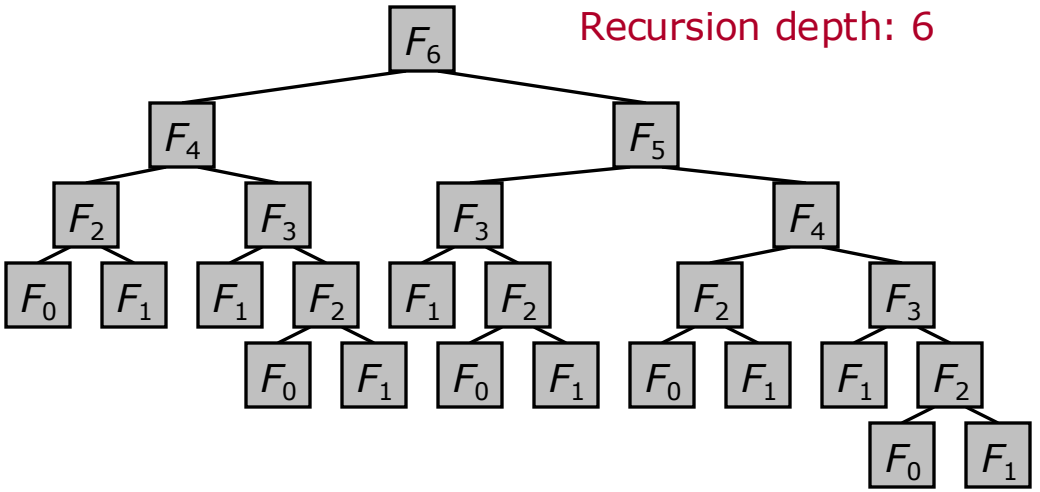
A function call's **recursion depth** is the greatest number of stack frames on the call stack *at any one time* as a result of the call.

`fibo_recurse.cpp`



Recursion depth: 8

`fibo_first.cpp`



Recursion depth: 6

When analyzing *time* usage, the total number of calls is of interest.

When analyzing *space* usage, the recursion depth is of interest.

Recursion vs. Iteration
Drawbacks of Recursion

Two factors can make recursive code inefficient, compared to iterative code.

- Inherent inefficiency of some recursive algorithms
  - But there are efficient recursive algorithms.
- Function-call overhead
  - Making all those function calls requires work: pushing and popping stack frames, saving return addresses, creating and destroying automatic variables.

These two are important regardless of the recursive algorithm used.

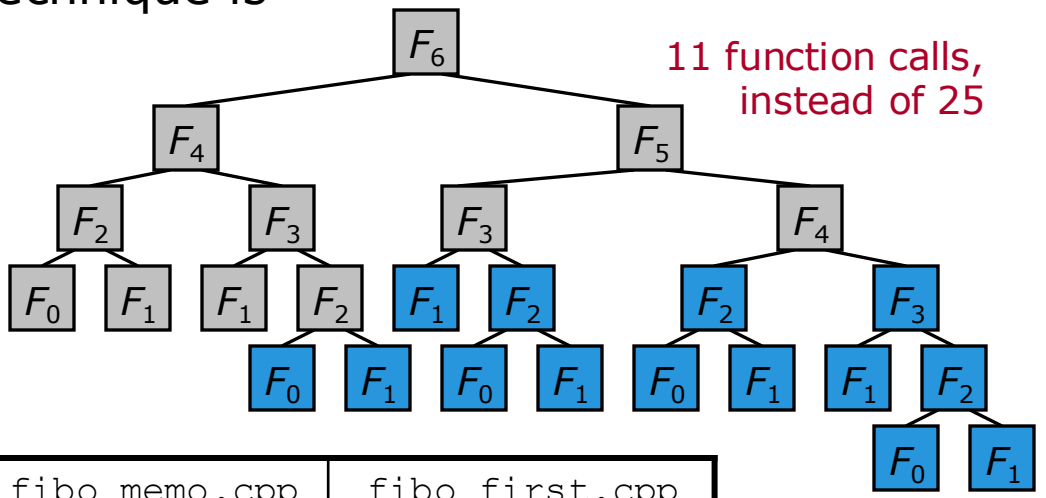
And recursion has another problem.

- Memory-management issues
  - A high recursion depth causes the system to run out of memory for the call stack. This is **stack overflow**, and it generally cannot be dealt with using normal error-handling procedures. The result is usually a crash.
  - When we use iteration, we can manage memory ourselves. This can be more work for the programmer, but it also allows proper error handling.

Dynamic programming (which does *not* mean what it sounds like) can greatly speed up some recursive algorithms.

- We save the results of computations, to avoid repeating them.
- In some contexts, this technique is called **memoizing**.

Apply this idea to `fibo_first.cpp`. **Blue** recursive calls are no longer necessary.



11 function calls, instead of 25

Fibonacci No.	`fibo_recurse.cpp`	`fibo_memo.cpp`	`fibo_first.cpp`
F_{10}	12 calls	19 calls	177 calls
F_{20}	22 calls	39 calls	21,891 calls
F_{40}	42 calls	79 calls	331,160,281 calls

Dynamic programming is covered in CS 411.

See `fibo_memo.cpp`.

Recursion vs. Iteration
Fibonacci Yet Again — Formula

There is a simple formula for F_n, using non-integer computations.

Let $\varphi = \frac{1+\sqrt{5}}{2} \approx 1.6180339$. (This is often called the **golden ratio**.)

For each nonnegative integer n, F_n is the nearest integer to $\frac{\varphi^n}{\sqrt{5}}$.

Here is `fibo` using this formula:

A floating-point literal with an "`L`" added at the end is of type `long double`.

```
bignum fibo(int n)
{
    long double phi = (1.0L + sqrt(5.0L)) / 2.0L;
    long double near_fibo = pow(phi, n) / sqrt(5.0L);
    // Our Fibonacci number is the nearest integer
    return bignum(near_fibo + 0.5L);
}
```

See `fibo_formula.cpp`.

Recursion vs. Iteration
Fibonacci Yet Again — Very Fast

An even faster method of computing Fibonacci numbers relies on the following facts:

- $F_{2n-1} = (F_{n-1})^2 + (F_n)^2$.
- $F_{2n} = 2F_{n-1}F_n + (F_n)^2$.

For the fast methods we mentioned earlier, computing F_n requires something like n arithmetic operations. But using the above facts, we can compute F_n using something like log n arithmetic operations—much less, when n is large.

This allows for easy computation of Fibonacci numbers that are much larger than any C++ built-in integer type can hold. To illustrate the power of this method, I have implemented it in Python, which has a built-in arbitrarily large integer type.

See `fibo_fast.py`.

Fibonacci Yet Again — Comments

A single problem may be solvable by many different methods.

- Different methods can have very different performance characteristics.
- It is possible that a very efficient method is not at all obvious.

Computing Fibonacci numbers is not something we need to do very often, in practice. But the above observations apply to other problems as well.

Next we will return to the problem of finding a key in a list.