What to know about matrix norms: Complete List!

- Matrix norms have all vector norm properties:
  \[ \|A\| = 0 \iff A = 0, \|A + B\| \leq \|A\| + \|B\|, \|\alpha A\| = |\alpha| \|A\| \]

- Only four norms in widespread use: \( \|\cdot\|_1 \), \( \|\cdot\|_2 \), \( \|\cdot\|_\infty \), and \( \|\cdot\|_{\text{Frob}} \)

- Three have computable formulas (1, \( \infty \), Frob)

- Three are induced from vector norms (1, 2, \( \infty \))

- All four have \( \|AB\| \leq \|A\| \|B\| \) (but—weirdly—for different reasons)

- Always \( \rho(A) \leq \|A\| \) for any norm, but learn to expect \( \rho(A) < \|A\| \)

- Iteration \( v, Av, A^2v, A^3v, \ldots \) converges if and only if \( \rho(A) < 1 \)

- Thus: if \( \|A\| < 1 \) then convergence \ldots but not conversely

- \( \|\cdot\|_2 \) is best for hermitian \( A \): if \( A^* = A \) then \( \rho(A) = \|A\|_2 \)

- Geometric picture clearest for \( \|\cdot\|_2 \): image under \( A \) of unit ball is ellipsoid with \( \|A\|_2 \) the length of the semimajor axis

- If \( A \) is square: \( \text{cond}(A) = \|A\|_2 \|A^{-1}\|_2 \)