ANSWERS TO EXAM III
BUELER: MATH 200 FALL 2001

1. A. increasing on \((-\infty, -1/2)\) and \((1/2, \infty)\); decreasing on \((-1/2, 0)\) and \((0, 1/2)\);
B. concave down on \((-\infty, 0)\) and concave up on \((0, \infty)\).

2. The linearization version: if \(f(x) = \sqrt{x}\) and \(a = 25\) then 
   \[L(x) = f(a) + f'(a)(x-a) = 5 + (1/10)(x-25).\]
   Then \(f(26) \approx L(26) = 5 + (1/10) = 5.1.\)

   The differentials version: if \(f(x) = \sqrt{x}\) then 
   \[d f = \frac{1}{2\sqrt{x}} \, dx.\]
   With \(x = 25\) and \(dx = 26 - 25 = 1\), then 
   \[d f = \frac{1}{2\cdot5} = \frac{1}{10}.\]
   And then \(f(26) \approx f(25) + d f = 5 + (1/10) = 5.1.\)

3. A. \(2t^{1/2} + (2/3)t^{3/2} + C\); B. \(\theta + C\).

4. A. The time from point A to D is \(\sqrt{1 + x^2}/(1) = \sqrt{1 + x^2}\) because the rate is 1 (miles per hour). Then time from point D to point B is \((6 - x)/6 = 1 - x/6\) because the rate is 6 (miles per hour). Then the total time is \(T(x) = \sqrt{1 + x^2} + 1 - x/6.\)

   B. Solve 
   \[T'(x) = \frac{x}{\sqrt{1 + x^2}} - \frac{1}{6} = 0.\]
   (Solve by cross-multiplying and squaring.) Get \(x = \sqrt{1/35}\)—which is very close to 1/6. Note \(T(0) = 2, T(\sqrt{1/35}) = 1 + \sqrt{35}/6, T(6) = \sqrt{37}\). Since \(\sqrt{35} < 6\), these numbers show \(T(\sqrt{1/35})\) is the smallest.

5. A. \(s(t) = -16t^2 + 10t + 4.\)

   B. You need both the velocity and the position at a certain time. For instance, you might have \(s(0)\) and \(v(0)\) as particular numbers.

6. A. Solve \(f'(x) = 4x^3 - 4x = 0\). Get \(x = -1, 0, 1\) as the only critical points.

   B. One way is to evaluate \(f''(x)\) at each of the critical points. Since \(f''(x) = 12x^2 - 4\), and \(f''(-1) = 8 > 0\), \(f''(0) = -4 < 0, f''(1) = 8 > 0\), we see that \(x = -1\) and \(x = 1\) are local minima (because the graph is concave up).

7. Newton’s method comes down to the formula: \(x_{n+1} = x_n - f(x_n)/f'(x_n).\) Calculate from \(x_0 = 0\) to \(x_1 = -1/3\) to \(x_2 = -1/3 + 1/90.\)

Extra Credit. Note \(x_1\) and \(x_2\) differ by 1/90. This is very close to 0.01 = 1/100. We know from previous examples that once Newton’s method gets close it converges fast to the solution. Thus based just on the values of \(x_1\) and \(x_2\) we guess that we are within .01.