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Dirichlet heat kernel in the exterior of a compact set.

Let $M$ be a complete, noncompact Riemannian manifold. Consider the behavior of the heat flow for $\Omega = M \setminus K$, where $K$ is a compact set. The authors establish the following result which relates the heat kernel $p(t, x, y)$ on $M$ and the Dirichlet heat kernel $p_\Omega(t, x, y)$ on $\Omega$:

**Theorem.** Suppose $M$ has nonnegative Ricci curvature. If the Brownian motion on $M$ is transient, then there exist $c, C > 0$ such that

$$cp(Ct, x, y) \leq p_\Omega(t, x, y) \leq p(t, x, y)$$

for all $t > 0$ and $x, y$ sufficiently far from $K$. If, however, $M$ is recurrent and a special connectedness-at-infinity condition holds (implying $M$ has only one end), then

$$c_1 D(t, x, y)p(C_1 t, x, y) \leq p_\Omega(t, x, y) \leq c_2 D(t, x, y)p(C_2 t, x, y),$$

where $D$ is a specified symmetric function defined in terms of the distance from $K$ and the volume growth of $M$. In the recurrent case, $\inf_{t>0} D(t, x, y) = 0$ for any $x, y \in M$.

This theorem gives a natural relation between the recurrence/transience of the Brownian motion and the heat absorption of $K$. Even for the Euclidean plane, a recurrent case, the result is apparently new: Example. If $M = \mathbb{R}^2$, $K$ is the closed unit disc, then

$$D(t, x, y) = \frac{\log |x| \log |y|}{(\log(1 + \sqrt{t}) + \log |x|)(\log(1 + \sqrt{t}) + \log |y|)}.$$

The proof of the recurrent case actually uses the transient result.

This interesting situation is possible because all results are computed for weighted manifolds. In particular, given appropriate $K$, recurrent $M$ support harmonic functions $h$, the weighted manifolds for which are transient. Also, the technique of proof involves the parabolic Harnack inequality of P. Li and S.-T. Yau [Acta Math. 156 (1986), no. 3-4, 153–201; MR 87f:58156], instead of the direct use of the curvature hypothesis. Thus the results apply in somewhat greater geometric generality.

This paper is part of a project by its authors to determine the behavior of the heat kernel on the ends of complete manifolds. Partial results are given in this direction.  

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[References]


