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On the microlocal regularity of the Schrödinger kernel.
(English. English summary)


This article is primarily an exposition of the results of the otherwise imposing paper by Craig, T. Kappeler, and W. Strauss [Comm. Pure Appl. Math. 48 (1995), no. 8, 769–860; MR 96m:35057]. That paper generalizes the following smoothing property of the classical free Schrodinger equation: The solutions of $i\partial_t \psi = -\Delta \psi$ on $\mathbb{R}^n$ tend to be smooth if the initial states $\psi_0$ have finite moments. In fact, if $\int |x^k \psi_0(x)|^2 dx < \infty$ for all $k \geq 0$ then the solution $\psi(x, t)$ is $C^\infty$ for all $t \neq 0$, which follows from the formula for the fundamental solution of the Schrodinger equation, for instance. The smoothing remains if a potential $V(x)$ with growth $\langle x \rangle^p$ for $p < 2$ is added to $-\Delta$. (The Mehler formula for the kernel of the harmonic oscillator shows that the case $p = 2$ is indeed critical.)

The above-mentioned paper by Craig, Kappeler and Strauss extends these results to variable coefficient, selfadjoint, asymptotically flat operators $-\partial_x (a^{ij}(x) \partial_x ) + V(x)$ on $\mathbb{R}^n$ with $p < 1$ in the above estimate for $V$. A certain quasiclassical property of the symbol $a(x, \xi) = a^{ij}(x) \xi_j \xi_l$ must hold: the solution $(x(s), \xi(s))$ of the Hamiltonian system $x_s = \partial_x a(x, \xi)$, $\xi_s = -\partial_x a(x, \xi)$ starting at $(x_0, \xi_0)$ escapes backward to infinity (i.e. $|x(s)| + |\xi(s)| \to \infty$ as $s \to -\infty$). The smoothing property then holds (“microlocally”) at $(x_0, \xi_0)$. A new symbol class, closely related to that of Hormander’s Weyl calculus, appears in the analysis. The article under review includes an examination of this class.

{For the entire collection see MR 98e:35003.}