Assignment # 3
Due Friday 10/3 at start of class

IV. Prove the expression for $\infty$-norm given on page 21 of Trefethen&Bau. That is, if $A$ is an $m \times n$ matrix then show that $\|A\|_\infty$ is the maximum row (absolute value) sum:

$$\|A\|_\infty = \max_{1 \leq i \leq m} \sum_{j=1}^{n} |a_{ij}|.$$ 

V. Let

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 5 & 8 \\ 5 & 8 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & 1 & -1 \\ 3 & 5 & 7 \\ 4 & 19 & 1 \end{bmatrix}.$$ 

(a) Use the result of problem IV above, and a formula on page 21, to determine $\| \cdot \|_1$ and $\| \cdot \|_\infty$ of $A$ and $B$.

(b) Use MATLAB to determine the norms $\| \cdot \|_1$, $\| \cdot \|_2$, and $\| \cdot \|_\infty$ of $A$ and $B$. Also determine $\rho(A)$, $\rho(B)$. Note any coincidences.

Exercise 3.1.

VI. Use MATLAB to reproduce the unit ball pictures on page 18, that is, of the unit balls in $\mathbb{R}^2$ for $\| \cdot \|_1$, $\| \cdot \|_2$, $\| \cdot \|_4$, and $\| \cdot \|_\infty$. Also do $\| \cdot \|_{1/2}$. Show the MATLAB input for at least one such ball. (Please do play around with MATLAB enough to make the pictures small and similar in style to the book.)

Exercise 3.3.

Extra Credit:

VII (Extra Credit). Estimate $\|A\|_4$, $\|B\|_4$ for $A$, $B$ in problem IV above.