Assignment #5
Due Monday March 7, 2005.

1. Read sections 2.13, 2.14, 2.15, and 2.16.

2. Exercise 2.6 (page 57).

3. Show that the explicit scheme

\[ \frac{U_{j}^{n+1} - U_{j}^{n}}{\Delta t} = \frac{(U_{j+1}^{n} - U_{j}^{n})p_{j+1/2} - (U_{j}^{n} - U_{j-1}^{n})p_{j-1/2}}{\Delta x^2} \]

for the differential equation

\[ \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left( p(x) \frac{\partial u}{\partial x} \right) \]

is consistent assuming \( p(x) \) has continuous derivative. Note \( x_{j+1/2} = (x_j + x_{j+1})/2 \) and \( p_{j+1/2} = p(x_{j+1/2}) \).

*Hint:* You seek the leading terms in the truncation error. Use Taylor’s theorem to get

\[ p(x + \epsilon) [u(x + \Delta, t) - u(x, t)] = (p(x) + p'(\xi_1)\epsilon) \left[ u_x(x, t)\Delta + \frac{1}{2} u_{xx}(x, t)\Delta^2 + \frac{1}{6} u_{xxx}(x, t)\Delta^3 + \frac{1}{24} u_{xxxx}(\xi_2, t)\Delta^4 \right]. \]

Now use this to compute

\[ p(x + \Delta x/2) [u(x + \Delta x, t) - u(x, t)] - p(x - \Delta x/2) [u(x, t) - u(x - \Delta x, t)], \]

and consider \( \lim_{\Delta x \to 0} \) of this quantity divided by \( \Delta x^2 \). This easier exercise replaces Exercise 2.7, page 57.

4. Exercise 2.9 (pages 57–58).