Assignment #3
Due Tuesday, 14 February 2006.

I. Read the “86. Linear fractional transformations” excerpt from Churchill & Brown. See also the “Appendix 2. Table of transformations of regions” excerpt. These supplement the rather sparse treatment of conformal maps in sections 20.8 and 20.9.

II. Do exercises:
Exercise E. Find a linear fraction transformation that maps the points $z_1 = \infty, z_2 = i, z_3 = 0$ to the points $w_1 = 0, w_2 = i, w_3 = \infty$.

Exercise F. Find a linear fraction transformation that maps the points $z_1 = 2, z_2 = i, z_3 = -2$ to the points $w_1 = 1, w_2 = i, w_3 = -1$. What is the image of the line $y = x$ (in the input $z$ space) under this transformation?

Exercise G. (a) Show that the composition of two linear fractional transformations is a linear fractional transformation.
(b) Let $T(z) = (az + b)/(cz + d)$ where $ad - bc \neq 0$. Show that $T^{-1} = T$ if and only if either $T(z) = z$ or $d = -a$. [Hint: Write the equation $T^{-1}(z) = T(z)$ as $(a + d)[cz^2 + (d - a)z - b] = 0$.]

III. Do exercises from RILEY, HOBSON, & BENCE:
20.9, 20.12