Assignment #8

Due Monday 5 December, 2011 at the start of class

Read subsections 7.1, 7.2, 2.6, and 7.3 of the text[1] quite carefully. Also lightly read subsections 5.7 and 5.8.1 of the text. (These are about Romberg integration, which I presented in-class in a very different way.) Then do the exercises below.

Page 409, Exercise 1. (*Hints*: 1. You are proving two directions: [if a triangular matrix is nonsingular then all its diagonal elements are nonzero] and [if a triangular matrix has only nonzero diagonal elements then it is nonsingular]. 2. Earlier you saw *how* to solve a triangular system. Use that to show one of the directions.)

Page 409, Exercise 2. (*Hint*: You can use the result in the previous problem.)

Page 79, Exercise 3.

Page 79, Exercise 4.

Page 416, Exercise 2.

Page 416, Exercise 7.

Page 417, Exercise 8.

P11. Use Gaussian elimination with partial pivoting *by hand* to solve the following system:

$$\begin{bmatrix} 3 & 5 & 7 \\ 8 & 1 & 6 \\ 4 & 9 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 2 \\ -2 \end{bmatrix}$$

Show the intermediate stages, after each row operation is completed.

P12. Use this pair of online codes

`http://www.dms.uaf.edu/~bueler/trap.m`
`http://www.dms.uaf.edu/~bueler/romberg.m`

to do Romberg integration on the problem

$$\int_1^3 x e^{-2x^2}\,dx$$

with a total of 64 subintervals; you will need to slightly modify the codes to do this. Evaluate the error . . . you need to compute the exact integral for this. Now modify the codes as needed to show the points in the h^2-versus-$T_n(f)$ which were extrapolated to zero to get the Romberg result. Evaluate the error from the 64 subinterval trapezoid result (which was used along the way). Explain/show the modifications you made.

[1] J. Epperson, *An Introduction to Numerical Methods and Analysis*, rev. ed., 2007.