Study Guide for Final Exam

the Final Exam is 10:15 am – 12:15 pm on Friday, 17 December

(*references are to 9th ed. Burden&Faires*)

($*$) = *I will supply but be able to apply!*

OVERVIEW. No calculator is needed. No notes. No book. Roughly 65% of the problems will be on post-midterm material (A # 6,7,8,9) while roughly 35% will be on earlier material (A # 1,2,3,4,5).

Problems will be in these categories:

- apply an **algorithm or method** in a simple concrete case
- **state** a theorem or definition
- **apply** a theorem or check a definition in a particular case
- **write** a short **pseudocode**, or a MATLAB/OCTAVE **code**, to state an algorithm
- **explain/show** in words: write in complete sentences (*What are the ideas behind some algorithm/method?* or *Why is one theorem or method better than another, when applied to some example?*)

ALGORITHMS AND METHODS. You must know what problem the algorithm/method solves, how to do a few steps or an easy case, and relative strengths and weaknesses:

- bisection method (*Alg 2.1*)
- Newton's method (*Alg 2.3*)
- Secant method (*Alg 2.4*)
- methods for finding interpolating polynomial $P(x)$ (*online slides*):
  - Vandermonde matrix
  - Newton form & matrix
  - Lagrange polys (*Thm 3.2*)
- Neville's method evaluates $P(x)$ w/o coeffs (*Thm 3.5; Alg 3.1;* `nev.m` *online*)
- Horner's method
- piecewise-linear interpolation
- natural cubic splines (*Alg 3.4*)
- Trapezoid, Simpson's, and Midpoint numerical integration rules (*section 4.3*)
- composite versions of above (*section 4.4, including Alg 4.1*)
- Romberg integration (*Alg 4.2*)
- Gaussian (= Gauss-Legendre) quadrature (*section 4.7*)
- Gaussian elimination and backward substitution (*Alg 6.1 and* `gebueler.m` *online*)
- Gaussian elimination using partial pivoting (*Alg 6.2*)
- forward substitution (*problem #7 on A # 9*)

DEFINITIONS.

- rate of convergence of sequences; "big O" (*p. 37*)
- order of convergence of sequences, including "linear" and "quadratic" convergence (*p. 79*)
- natural cubic spline
- degree of (accuracy/) precision
- Legendre polynomials (*pp. 230–231*)

THEOREMS.

- Intermediate Value Theorem (*Thm 1.11*)
- Mean Value Theorem (*Thm 1.8*) and Rolle's Theorem (*Thm 1.7*)
- Taylor's Remainder Theorem (*Thm 1.14*)
- Lagrange's Remainder Theorem (*Thm 3.3*)
- fixed-point theorems:
  - existence and uniqueness of fixed point of $g(x)$ (Thm 2.3)
  - iterations $p_n = g(p_{n-1})$ converge if [*some conditions*] (*Thm 2.4*)
- Newton's method converges quadratically (*Thm 2.9*)
- error theorem for piecewise-linear interpolation (*on A#6;* $*$)
- error theorems for Trapezoid, Simpson's, and Midpoint numerical integration rules (*section 4.3;* $*$)
- error theorems for composite versions of above (*section 4.4;* $*$)
- degree of precision of Gaussian quadrature (*Thm 4.7*)
- operation count for Gaussian elimination is essentially $(2/3)n^3$ (*section 6.1*)

OTHER.

- be able to sketch the basic ideas of 64-bit binary representation of real numbers; "IEEE 754" (*p. 17–19*)
- be able to count operations or function evaluations in a single, or two nested, `for` loops