Assignment #9

Due Monday 13 December at start of class.

Read sections 4.7, 6.1, 6.2, and 6.3 of the textbook Burden & Faires.

1. Write two short MATLAB/OCTAVE functions, with care to make them as simple and clear as possible, which do Gaussian quadrature using the $n = 2$ and $n = 3$ rules, respectively. The first lines of these functions should be

```
function z = gaussian2(f,a,b)
function z = gaussian3(f,a,b)
```

respectively. Note that online example

```
http://www.dms.uaf.edu/~bueler/polyintcheb.m
```

has the same kind of inputs and outputs, and can be a model to follow. Your programs will use the shift-and-scale technique which converts Gaussian quadrature on $[-1, 1]$ to an arbitrary interval $[a, b]$.

Test your programs on the integral

$$\int_1^2 xe^{-x}\, dx$$

and compare to the exact answer.

2. Recall $P_3(x) = (1/5)(5x^3 - 3x)$. For $P(x) = x^5 + x^4 - x^3 - x^2 - x + 17$, compute $Q(x)$ and $R(x)$ so that $P(x) = Q(x)P_3(x) + R(x)$. Show all steps, and do this by-hand. What are the degrees of $Q(x)$ and $R(x)$? Explain why these degrees, or at least upper bounds on these degrees, are known in advance before doing the calculation.

3. Write a short MATLAB/OCTAVE script that uses Newton's method to find the roots of

$$P_5(x) = (1/63)(63x^5 - 70x^3 + 15x).$$

Start by graphing $P_5(x)$ to get estimates of initial guesses. Explain why you really only need to use Newton's method two times, not five times, to do this problem. Relate the results to Gaussian integration.

4. For this system of equations, first graph each equation as a line and thereby solve the system geometrically. Then use Gaussian elimination with backward substitution to find the solution. Do this by hand.

$$x_1 + 2x_2 = 0,$$
$$x_1 - x_2 = -3.$$

5. Given the linear system

$$\begin{aligned} x_1 - x_2 + \alpha x_3 &= -2, \\ -x_1 + 2x_2 - \alpha x_3 &= 3, \\ \alpha x_1 + x_2 + x_3 &= 2. \end{aligned}$$

(a) Find value(s) of α for which the system has no solutions. Also describe in words the geometrical situation in which the system has no solutions.

(b) Find value(s) of α for which the system has infinitely-many solutions. Also describe in words the geometrical situation in which the system has infinitely-many solutions.

(c) Assuming that a unique solution exists for a given α, find the solution.

Hint on all parts: Leaving α undetermined, you can do Gauss elimination with backward substitution. You should then consider the meaning of the various divisions. Also, you can check your result in part **(c)** for specific values of α using MATLAB/OCTAVE.

6. Perform the following matrix-matrix multiplications *by-hand*:

(a)

$$\begin{bmatrix} -2 & 3 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -5 & 2 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 3 & -1 & 0 \\ 2 & -2 & 3 \\ -2 & 1 & 4 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 4 & -1 \\ 3 & -5 \end{bmatrix}$$

7. Consider this general *lower triangular* system of linear equations:

$$\begin{aligned} &a_{11}x_1 && = b_1 \\ &a_{21}x_1 + a_{22}x_2 && = b_2 \\ &a_{31}x_1 + a_{32}x_2 + a_{33}x_3 && = b_3 \\ &\quad\vdots \qquad\qquad\qquad \ddots && \quad\vdots \\ &a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n && = b_n \end{aligned}$$

(a) Carefully describe a pseudocode, or write a working MATLAB/OCTAVE program, which does "forward substitution" to solve these equations. What assumptions are necessary to get a unique solution?

(b) Count the number of additions, subtractions, multiplications, and divisions in the algorithm in part **(a)**, giving the count for each type of operation separately.