Assignment #8

Due Monday 29 November at start of class.

Read sections 4.5 and 4.7 in the textbook (Burden & Faires, 9th ed.).

1. (*Do this "by-hand", using* MATLAB/OCTAVE *as a calculator when needed, but showing major steps.*) Use Romberg integration to compute $R_{3,3}$ for the integral

$$\int_0^1 x^2 e^{-x}\,dx.$$

Compare to the exact value.

2. Write a MATLAB/Octave program that implements Algorithm 4.2, Romberg integration. The inputs should be f (a function), limits a and b, and an integer n, which is the number of rows of the Romberg table ($R_{k,j}$) to compute. The output is the Romberg approximate integral $R_{n,n}$, an estimate of the integral $\int_a^b f(x)\,dx$. Demonstrate your routine on the integral

$$\int_{-1}^{2} (\cos x)^3\,dx$$

using $n = 3$ and $n = 7$, and give the actual errors. Finally, in the $n = 3$ case show the whole triangular Romberg table $\{R_{k,j}\}$ which your program generates. (*Your program generates the Romberg table row-by-row, so you can just have the program print out the rows as it goes. But make it clear to me which row is which.*)

Extra Credit. Write a version of the above routine which accepts a *tolerance TOL*, instead of n, and runs the Romberg method until some accuracy estimate is satisfied. Explain your accuracy estimation method in a few complete sentences, e.g. as documentation in the program.

3. (*An extrapolation method for intepolation. Like Romberg, but not for the same purpose.*) Consider the problem of interpolating $f(x) = \cos(3x)$ on the interval $[0, 4]$. Let $h = (4-0)/N$, and consider equally-spaced N degree (thus $N+1$ point) polynomial interpolation of $f(x)$. More specifically, consider using these polynomials $P_N(x)$ to approximate $f(\pi) = -1$.

(a) Show, on h-versus-$P_N(\pi)$ axes, a plot of the successive approximations $P_N(\pi)$ to $f(\pi)$. Use $N = 1, 2, 4, 8, 16$.

(b) Think of $P_N(\pi)$ as a function of h: $F(h) = P_N(\pi)$. Use the data in part **(a)** to extrapolate this function to zero to get $F(0) = P_\infty(\pi)$. How accurate is your result?

(c) Speculate/comment on whether this method is promising for interpolation. How efficient is it? (*Compared to what?*) How could you know in advance how accurate it is?

Extra Credit. Repeat the above problem but do it with Chebyshev spacing. (*You will have to think about the meaning of "h".*)

4. Approximate the following integrals using Gaussian quadrature with $n = 2$, and compare your results to the exact values of the integrals.

(a) $\displaystyle\int_1^{1.5} x^2 \ln x \, dx,$ **(b)** $\displaystyle\int_1^{1.6} \frac{2x}{x^2 - 4} \, dx$

5. Repeat the above exercise with $n = 3$ Gaussian quadrature.

6. Determine constants a, b, c, d that will produce a quadrature formula

$$\int_{-1}^{1} f(x) \, dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

that has degree of precision 3.