Assignment #7

Due Wednesday 17 November at start of class.

Read sections 4.3, 4.4, and 4.5 in the textbook (Burden & Faires, 9th ed.).

1. Approximate the following integrals by the (non-composite) Trapezoid, Simpson's, and Midpoint rules:

(a) $\displaystyle\int_0^{0.5} (\cos x)^2\,dx$ **(b)** $\displaystyle\int_{-0.5}^{0} x\ln(1+x)\,dx$

2. Using the error theorems in the text, and proven in class, estimate the error in the using the above rules to do the integrals in problem **1**. Using the fact that the exact value for the integral in part **(a)** is 0.460367746, state the actual errors for part **(a)**.

3. The Trapezoid rule applied to some integral $\int_0^2 f(x)\,dx$ gives the value 5, and the Midpoint rule gives the value 4. What value does Simpson's rule give?

4. Find the degree of precision of the quadrature formula

$$\int_{-1}^{1} f(x)\,dx \approx f\left(-\frac{\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right).$$

5. Use the composite Trapezoid and Simpson's rules, with $n = 10$, to approximate the integral

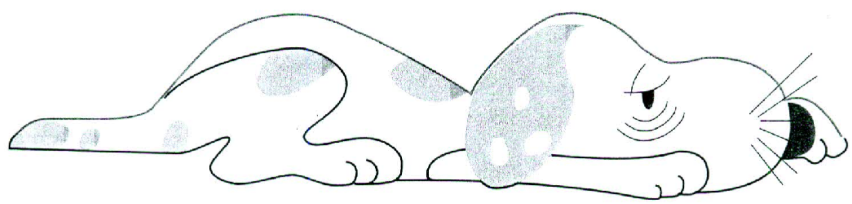
$$\int_0^{0.5} (\cos x)^2\,dx.$$

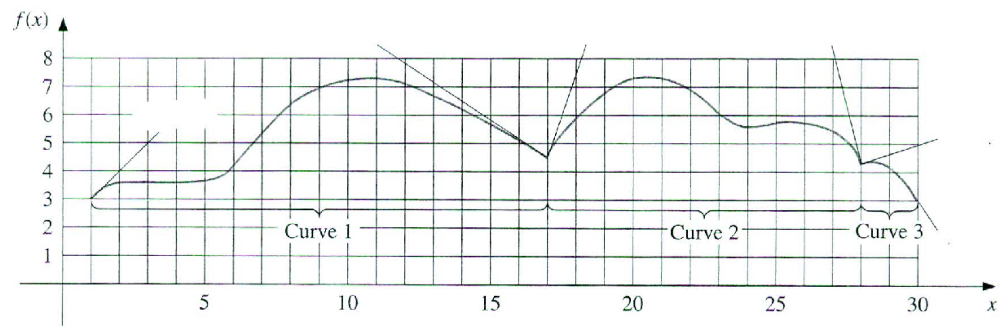
Compute the actual error, noting again that the exact value for the integral is 0.460367746.

6. Apply the error theorems in section 4.4, namely theorems 4.5 and 4.4, respectively, to estimate an n sufficient so that the Trapezoid rule and Simpson's rule approximate the integral in problem **5** to accuracy 10^{-13}.

Chose **ONE** of the following two problems **A** and **B**.

A. The upper portion of this noble beast is to be approximated using natural cubic spline interpolation. The curve was drawn on a grid from which the table was constructed. Use the online code `ncspline.m` to construct the three natural cubic splines. Plot them together on one graph so that your output *looks* like the upper portion.





Curve 1			Curve 2			Curve 3		
i	x_i	$f(x_i)$	i	x_i	$f(x_i)$	i	x_i	$f(x_i)$
0	1	3.0	0	17	4.5	0	27.7	4.1
1	2	3.7	1	20	7.0	1	28	4.3
2	5	3.9	2	23	6.1	2	29	4.1
3	6	4.2	3	24	5.6	3	30	3.0
4	7	5.7	4	25	5.8			
5	8	6.6	5	27	5.2			
6	10	7.1	6	27.7	4.1			
7	13	6.7						
8	17	4.5						

B. Using grid paper with approximately 1 cm or 5 mm spacing, draw an outline of your hand. Extract the coordinates of at least $N = 11$ points along this outline and make a table of (x, y) coordinates of these points. (About 20 points is recommended.)

Now treat these points as samples of functions $x(t)$ and $y(t)$. To do this you may use the count (index) of the points as the t-coordinate, like $t_j = j$, or you may choose your own scheme. Get N points (t_j, x_j) and another N points (t_j, y_j). Plot the two functions $x = x(t)$ and $y = y(t)$ on the same axes, that is, plot these data points. Now use `ncspline.m` to compute the natural cubic splines interpolating these data, one for $x(t)$ and one for $y(t)$. Finally, plot just the resulting smooth curve in the (x, y) plane, "dropping" t-coordinates. It should be a smooth version of your hand!

Extra Credit. Improve on the above. In particular, write a program that uses more natural user input, such as mouse-click-location, or GPS coordinates from a mobile phone for that matter, and computes and draws a (t-parameterized) smooth curve through that data. Presumably that "smooth curve" is the natural cubic spline.

Extra Credit. Write a numerical integrator using the natural cubic spline.