Assignment #5

Due Friday, 22 October at the start of class

Read subsections 3.1 and 3.2 of the text, and do the following exercises.

1. Let $x_0 = 1, x_1 = 1.25, x_2 = 1.6$. For each of the following functions, construct the interpolating polynomial $P(x)$ of degree 2. Compute $f(1.4)$ and $P(1.4)$. Report the actual absolute error. (*I am not concerned with which method you choose to compute $P(x)$ in each part. But do show your work, so I know you have* some *method.*)

a. $f(x) = \sin(\pi x)$

b. $f(x) = \log_{10}(3x - 1)$

c. $f(x) = \sqrt[3]{x - 1}$

2. For each of the functions in problem **1**, and considering the same interpolation problems, use the Lagrange Remainder Theorem (Theorem 3.3) to find an error bound for the approximations. That is, find *bounds* on $|P(1.4) - f(1.4)|$ from the Theorem. (*Thus you should identify the parts as* **a**, **b**, **c**, *just as in problem* **1**. *Also note that your error bounds must exceed the actual errors!*)

3. Let $P_3(x)$ be the interpolating polynomial for the data $(0, 0), (0.5, y), (1, 3), (2, 2)$. The coefficient of x^3 in $P_3(x)$ is 6. Find y.

4. Use Neville's method to approximate $\sqrt{2}$ with the following functions and values:

a. $f(x) = 2^x$ and the values $x_0 = -2, x_1 = -1, x_2 = 0, x_3 = 1, x_4 = 2$.

b. $f(x) = \sqrt{x}$ and the values $x_0 = 0, x_1 = 1/3, x_2 = 1, x_3 = 3, x_4 = 4$.

c. Compare the actual accuracy of the approximations in parts **a** and **b**. (*This assumes you can find an exact value for* $\sqrt{2} \dots$)

5. In probability and statistics there are many uses for the "error function"

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt.$$

Unfortunately, it is known that there is no elementary formula for the antiderivative of $f(t) = \exp(-t^2)$, so the above formula for $\text{erf}(x)$ cannot be simplified. This problem describes one method for approximating $\text{erf}(x)$ with known accuracy anyway.

a. Let $P(t)$ be the degree 8 polynomial interpolant of $f(t) = \exp(-t^2)$ using the points $x_j = 0.1j$ for $j = 0, \dots, 8$. That is, $x_0 = 0.0, x_1 = 0.1, x_2 = 0.2, \dots, x_8 = 0.8$. Compute the coefficients of $P(t)$ in a program. The Vandermonde method is almost certainly the easiest way to get these coefficients.

b. Use the Lagrange Remainder Theorem (Theorem 3.3) to find a bound for the approximation error $\max |P(t) - f(t)|$ on the interval $t \in [0, 0.8]$.

c. Now, unlike $f(t)$, you do know how to integrate $P(t)$. I.e. you know how to find an antiderivative $Q(t)$ of $P(t)$: $Q'(t) = P(t)$. Do so, and then use the Fundamental Theorem of Calculus to approximate $\text{erf}(0.8)$. That is, compute

$$F(0.8) \equiv \frac{2}{\sqrt{\pi}} \left(Q(0.8) - Q(0)\right) \approx \text{erf}(0.8).$$

(*Note that the* MATLAB/OCTAVE *program you wrote in part* **a** *can be amended to compute the antiderivative and then the value of* $\text{erf}(0.8)$. Think *before either writing code or doing the integral yourself using nasty decimal expansions of the coefficients.*)

d. On the other hand, `erf` is a built-in function in MATLAB/OCTAVE. Use it to compute the actual absolute error in your approximation of $\text{erf}(0.8)$.

e. Use the bound computed in part **b** to give a bound on the error $|F(0.8) - \text{erf}(0.8)|$. (*This bound should not require any knowledge gained in part* **c**, *except that, as usual, the bound must be greater than the actual erro.*)