Assignment #3

Due Friday 1 October at start of class.

These exercises are not based on the course textbook (Burden & Faires). Instead, you should read the online slides at

`http://www.dms.uaf.edu/~bueler/polybasics.pdf`

and perhaps also listen to the .mp3 voiceover for the slides at

`http://www.dms.uaf.edu/~bueler/polybasics.mp3`

1. Consider these three points: $\{(1,1),\,(3.5,7),\,(4,5)\}$. Find the polynomial $P(x)$ of degree 2 which passes through these points. Do this three different ways, by using

(a) the Vandermonde matrix method,

(b) the Newton form and its triangular matrix method, and

(c) the Lagrange form.

Comment. In the parts above you may use MATLAB/OCTAVE to do the computations, including solving linear systems using the backslash operation. Please make sure to show me the entries in the matrices in parts **(a)** and **(b)**, however. Also show me the Lagrange polynomials in part **(c)** before you combine them to give $P(x)$. You do not have to simplify your answers to the parts above; use the most convenient way to write down $P(x)$.

(d) Now show, by hand, that the polynomials in parts **(a)**, **(b)**, and **(c)** are all identical. For instance, you can put the answers from parts **(b)** and **(c)** in standard (monomial) form, as you presumably already did for part **(a)**.

2. Write a short MATLAB/OCTAVE program which generates 7 random points and then uses the MATLAB/OCTAVE command `vander` to construct the polynomial $P(x)$ through these points. Specifically, do this to generate the points:

```
>> x = randn(1,7)
>> y = randn(1,7)
```

Your program should display the polynomial in standard form, as a string. In a figure similar to the one on the "did we solve the problem?" slide, plot both the polynomial and the points it goes through.

Practical comment. Once you have generated the figure you can either print from the MATLAB figure window or (esp. for OCTAVE users) put it into a printable PDF by the command `>> print -dpdf myfigure.pdf`

3. **(a)** Solve this triangular system by hand:

$$\begin{aligned} 3x \quad\quad\quad\quad &= 9 \\ -x - 6y \quad\quad &= -3 \\ 2x + 5y - 3z &= 0 \end{aligned}$$

(b) Write down formulas for each component $x_i,\ i = 1, 2, \ldots, n,$ of the solution of the triangular linear system:

$$\begin{array}{lcl} a_{11}x_1 & = & b_1 \\ a_{21}x_1 + a_{22}x_2 & = & b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 & = & b_3 \\ \vdots \quad\quad \vdots & & \vdots \\ a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n & = & b_n \end{array}$$

4. Consider the function $f(x) = 2^x$ on the interval $[0.5, 3]$.

(a) Find the polynomial $P(x)$ of degree 5 which interpolates $f(x)$ at the 6 equally-spaced points $0.5, 1, 1.5, 2, 2.5, 3$. (*You may use any of the three methods, Vandermonde, Newton, or Lagrange. Your choice.*)

(b) Find the polynomial $Q(x)$ of degree 5 which interpolates $f(x)$ at the 6 *un*equally-spaced points $0.5, 0.7, 1.1, 1.5, 2, 3$. (*Again, use the method of your choice.*)

(c) Make a plot which shows $f(x)$, $P(x)$, and $Q(x)$ on the same axes. Between $P(x)$ and $Q(x)$, which is more accurate as an approximation of $f(x)$?

Comment. On all parts of this problem, use MATLAB/OCTAVE as effectively as you wish.