Assignment #2

Due Monday, 27 September at the start of class

Read subsections 1.3 and 2.1 of the text, and do the following exercises.

1. Find the rates of convergence of the following sequences as $n \to \infty$. In particular, write the answer in the form "$\alpha_n = 0 + O(1/n^q)$."

a.

$$\lim_{n\to\infty} \sin\left(\frac{1}{n}\right) = 0$$

b.

$$\lim_{n\to\infty} \sin\left(\frac{1}{n^2}\right) = 0$$

c.

$$\lim_{n\to\infty} [\ln(n+1) - \ln n] = 0$$

2. Suppose that $0 < q < p$. Suppose that $\alpha_n = \alpha + O(n^{-p})$ (as $n \to \infty$). Show that $\alpha_n = \alpha + O(n^{-q})$.

3. The Maclaurin series, which is the Taylor series with basepoint $x_0 = 0$, for the arctangent function converges for $-1 < x < 1$ and is given by:

$$\arctan x = \sum_{i=1}^{\infty} (-1)^{i+1} \frac{x^{2i-1}}{2i-1}.$$

a. Write a simple MATLAB/OCTAVE code that computes $P_{99}(0.3)$, that is, the value of the partial sum, at $x = 0.3$, which has highest power "x^{99}."

b. Note that $\tan(\pi/4) = 1$. Based on this, find out the first n for which

$$|4P_n(1) - \pi| \leq 10^{-3}.$$

Accomplish this *either* by actually doing the sums in MATLAB/OCTAVE, or by exploiting facts about alternating series.

For problems **4,5,6**, which are based on section 2.1, you are encouraged to write a "special-case" MATLAB/OCTAVE code. That is, a code similar to the one posted at the class webpage for Monday 13 September:

`http://www.dms.uaf.edu/~bueler/Math310F10.htm`

4. Use the bisection method to find solutions accurate to 10^{-2} for

$$x^4 - 2x^3 - 4x^2 + 4x + 4 = 0$$

on each interval.

a. $[-2, -1]$.

b. $[0, 2]$.

5. Use the bisection method to find a solution, accurate to 10^{-6}, for

$$3x - e^x = 0$$

on the interval $[1, 2]$.

6. **a.** Sketch the graphs of $y = x$ and $y = \tan x$. (Or use MATLAB/OCTAVE to make a plot.)

b. Use the bisection method to find an approximation to within 10^{-5} of the first positive solution of $x = \tan x$.