Math 310 Numerical Analysis, Fall 2010 (Bueler; created 9/3/2010; revised 9/7/2010)

Assignment #1

Due Monday, 13 September at the start of class $\longleftarrow$ *REVISED DUE DATE*

1. What is your major? Has anyone other than me (Bueler) ever suggested you learn MATLAB or OCTAVE?

$\longleftarrow$ *The answers to these questions should go in an email to Bueler* (`elbueler@alaska.edu`), *or in a note under his office door, before Wed. 8 September. The rest of the assignment, everything below, is due at the "Monday, 13 September" date.*

Next: Read subsections 1.1 and 1.2 of the text, and do the following exercises.

2. Show that the following equations have at least one solution in the given intervals:

a. $x\cos x - 2x^2 + 3x - 1 = 0$, $[0.2, 0.3]$ and $[1.2, 1.3]$

b. $(x-2)^2 - \ln x = 0$, $[1, 2]$ and $[e, 4]$

3. Find $\max_{a \le x \le b} |f(x)|$ for the following functions and intervals:

a. $f(x) = (4x-3)/(x^2-2x)$, $[0.5, 1]$

b. $f(x) = 2x\cos(2x) - (x-2)^2$, $[2, 4]$

4. Suppose $f \in C[a, b]$ and $f'(x)$ exists on (a, b). Show that if $f'(x) \neq 0$ for all x in (a, b), then there can exist at most one number p in $[a, b]$ with $f(p) = 0$.

5. Find the third Taylor polynomial $P_3(x)$ for the function $f(x) = \sqrt{x+1}$ about $x_0 = 0$. Approximate $\sqrt{0.5}$, $\sqrt{0.75}$, $\sqrt{1.25}$, and $\sqrt{1.5}$ using $P_3(x)$, and find the actual errors.

6. Let $f(x) = 2x\cos(2x) - (x-2)^2$ and $x_0 = 0$.

a. Find the third Taylor polynomial $P_3(x)$ and use it to approximate $f(0.4)$. Compute the actual error.

b. Use the error formula in Taylor's Theorem to find an upper bound for the error $|f(0.4) - P_3(0.4)|$. (*This upper bound does not require that you know the actual error. But your answer here is only correct if it is* greater than *the actual error. Similar advice applies to part* **d**.)

c. Find the fourth Taylor polynomial $P_4(x)$ and use it to approximate $f(0.4)$. Compute the actual error.

d. Use the error formula in Taylor's Theorem to find an upper bound for the error $|f(0.4) - P_4(0.4)|$.

7. Use the error term of a Taylor polynomial to estimate the error involved in using the approximation $\sin x \approx x$ to approximate $\sin 1^\circ$.

8. Compute the absolute error and relative error in approximations of p by p^*.

a. $p = \pi, \quad p^* = 22/7$

b. $p = e, \quad p^* = 2.718$

c. $p = 8!, \quad p^* = 40000$

d. $p = 8!, \quad p^* = \sqrt{2\pi}\, 8^{8.5}\, e^{-8}$

9. The number e can be defined by $e = \sum_{n=0}^{\infty}(1/n!)$, where we define (or agree that) $0! = 1$. Compute the absolute and relative error in the following approximations of e:

a. $\displaystyle\sum_{n=0}^{5} \frac{1}{n!}$ **b.** $\displaystyle\sum_{n=0}^{10} \frac{1}{n!}$

10. Use the 64-bit long real format to find the decimal equivalent of the following floating-point machine numbers:

a. `0 10000001010 1001001100000000000000000000000000000000000000000000`

b. `1 10000001010 1001001100000000000000000000000000000000000000000000`

c. `0 01111111111 0101001100000000000000000000000000000000000000000000`

d. `0 01111111111 0101001100000000000000000000000000000000000000000001`