Math 310 Numerical Analysis (Bueler; December 13, 2009)

Review for in-class Final Exam
(Friday, 18 December, 10:15–12:15, Gruening 202 as usual)

Earlier. First, see **Review for Midterm** sheet attached to A#5. All material on that sheet, and on the Midterm itself, is important, and potentially on the Final Exam. But Final will emphasize recent material. Here are some highlights from Midterm and earlier:

- Taylor's Theorem with (Lagrange) remainder (p. 6)
- "$\{x_n\}$ *converges at rate* α" (including *linearly*, *superlinearly*, and *quadratically*)
- bisection, Secant, and Newton's method, and the problem they solve: $f(x) = 0$
- how to compute matrix-vector and matrix-matrix products: Ax, AB
- what is A^{-1} ... and that the solution of $Ax = b$ can be written as $x = A^{-1}b$
- ... and this becomes "`>> x = A\b` " in Matlab/Octave
- what *back-substitution* is, what it solves, and that it requires $O(n^2)$ ops
- ... and same for *forward-substitution*
- how to do a few steps of Gauss elimination by hand

Chapter 4. numerical linear algebra (just an introduction!)

- explain how $A = LU$ decomposition is really Gauss elimination
- ... and know that it requires $O(n^3)$ operations
- what is role of pivoting in LU and Gauss?
- know how to do either partial or scaled row pivoting
- ... and that pivoting produces $PA = LU$
- know how to apply $A = LU$ to solve system $Ax = b$ by two triangular problems
- ... and know what changes with pivoting
- LU (Gauss elim) in the tridiagonal case: how-to, and how many operations?

Chapter 6. interpolation and function approximation

- Hörner's method for evaluating polynomials from their coefficients: why and how
- Vandermonde approach to finding n degree poly interpolant through $n + 1$ points
- Newton approach ... Lagrange approach ... and apply any of these on small example
- theorem stating that there is a unique poly intepolant (p. 309)
- theorem giving remainder for polynomial interpolation (p. 315), and apply it
- how-to, and error theorem (on A#7 solns) for piecewise-linear interpolation
- explain ideas behind cubic splines (why they are nice, why it works quickly)

Chapter 7. numerical integration

- idea of Newton-Cotes: interpolate integrand by polynomial and integrate that
- derive easy (e.g. midpoint or trapezoid) Newton-Cotes
- derive general formula for coefficients A_i in n degree Newton-Cotes
- prove general theorem for Newton-Cotes: $\left|\int_a^b f(x)\,dx - \sum_{i=0}^n A_i f(x_i)\right| \le \dots$
- what does "composite trapezoid", "composite Simpson's" mean?
- for composite trap. and Simp.: apply stated remainder formulas in easy cases
- mean value theorem for integrals (p. 19)
- explain ideas behind Romberg integration; also: how to minimize # of fcn evals?

Chapter 8. ordinary differential equations

- be able to state the general "standard prediction problem" (a.k.a. an ODE IVP)
- solve by hand the $\frac{dx}{dt} = F(t)$ and $\frac{dx}{dx} = G(x)$ cases
- state Euler's method in general and apply it in easy example
- how to build a Taylor method of any degree