Assignment #7

DUE *Friday 20 November, 2009*

Problems 6.1, exercise 13.

Problems 6.1, exercise 27.

Problems 6.1, exercise 37.

Exercise 1. (*There is no need in any part of this problem to compute the interpolants. Just estimate the errors.*) In class I gave an error formula for piecewise-linear interpolation. Theorem 2 on page 315 of the text gives the error for polynomial interpolation, of course. For parts **(a)**, **(b)**, and **(c)** below, estimate quantitatively the error in using n-degree polynomial interpolation and in using piecewise-linear interpolation, for the given $n+1$ nodes:

(a) $f(x) = e^{-x}$; nodes `x_i = 0:0.1:1`

(b) $f(x) = \sin(7x)$; nodes `x_i = 0:0.1:1`

(c) $f(x) = \sin(7x)$; nodes `x_i = 0:0.04:1`

(d) Comment on the results, and consider the general effect of "m" in interpolating $\sin(mx)$ by the two methods.

Exercise 2. Consider the problem of interpolating a polynomial to find its coefficients. Suppose that $p(x) = a_0 + a_1 x + \cdots + a_n x^n$ exactly, but that we know only the values $y_i = p(x_i)$ at $n+1$ distinct points $x_0, \ldots, x_n$, and *not* the coefficients a_j. So we use one of our schemes (Vandermonde, Newton, or Lagrange) to compute approximate coefficients $\tilde{a}_0, \ldots, \tilde{a}_n$. The general question is, *how close are the computed coefficients $\tilde{a}_j$ to the actual coefficients a_j?*

(a) Evaluate the polynomial $p(x) = (x-1)(x-2)(x-3)(x-4)$ at the points $x_0 = 0.1$, $x_1 = 0.9$, $x_2 = 1.4$, $x_3 = 3.14159$, $x_4 = 4.1$. Using the values $y_j = p(x_j)$, and the method of your choice implemented in MATLAB/OCTAVE, compute the coefficients $\tilde{a}_j$. Check your answer by computing the exact coefficients by hand, and give a quantitative estimate of the error in the computed coefficients.

(b) *The Wilkinson polynomial.* Let $x_j = j + 0.141$ for $j = 0, \ldots, 20$. Evaluate this formula for $P(x)$ at x_j to find $y_j = P(x_j)$: $P(x) = \prod_{i=1}^{20}(x-i)$. Using the method of your choice, on the values y_j, compute the approximate coefficients $\tilde{a}_0, \ldots, \tilde{a}_{20}$. Comment.

(c) The exact coefficients of $P(x)$ are in this code, which is online at

`http://www.dms.uaf.edu/~bueler/wilkinson.m`

Note that the coeffients from `wilkinson.m` are in a particular order, so be careful using them, but they *are* the exact coefficients. How close is your computed list from part **(b)** to these exact values? Explain what you can.

(extra credit) Somewhere between degree 4 and degree 20 a disaster occurred. At what degree did it become effectively-impossible to compute the coefficients of these "$\prod_i (x-i)$" polynomials?

For more information: *To help understand the disaster in* **(b)** *and* **(c)**, *see "Perfidious Polynomial" on pages 71–72 of the textbook, and/or see the page*

`http://en.wikipedia.org/wiki/Wilkinson's_polynomial`