Quiz # 6 Solutions.

1. I get

\[ \mathbf{v}(t) = -4 \sin t \mathbf{i} + 4 \cos t \mathbf{j} + 3 \mathbf{k}, \]

Thus

\[ |\mathbf{v}(t)| = \sqrt{4^2 + 3^2} = 5. \]

Thus

\[ s = \int_0^t 5 \, d\tau = 5t. \]

2. domain = \( \{(x, y) \mid x \neq 0\} \);

range = \((0, \infty)\).

3. I sketched the level surface \( 1 = f(x, y, z) \), that is \( 1 = y^2 + z^2 \). This is a cylinder which has radius 1 and has the \( x \)-axis along its axis.

4. I calculated:

\[ \mathbf{T} = -\left(\frac{4}{5}\right) \sin t \mathbf{i} + \left(\frac{4}{5}\right) \cos t \mathbf{j}, \]

\[ \frac{d\mathbf{T}}{dt} = \left(\frac{4}{5}\right) \cos t \mathbf{i} - \left(\frac{4}{5}\right) \sin t \mathbf{j}, \]

\[ \left| \frac{d\mathbf{T}}{dt} \right| = \frac{4}{5}, \]

and

\[ \kappa = \left| \frac{\frac{d\mathbf{T}}{dt}}{|\mathbf{v}|} \right| = \frac{4/5}{5} = \frac{1}{25} = \frac{1}{6.25}. \]

The last expression is close to the meaning. That is, the osculating circle of the curve, at any point, has radius 6.25. Since, in fact, the curve is a helix which follows a cylinder of radius 4, this makes sense.