Solutions to Quiz #8

1. By the Ratio test,

$$\lim_{n\to\infty} \frac{\frac{|x-2|^{n+1}}{(n+1)^2}}{\frac{|x-2|^n}{n^2}} = \lim_{n\to\infty} \frac{|x-2|^{n+1}\cdot n^2}{(n+1)^2\cdot|x-2|^n} = |x-2| \lim_{n\to\infty}\frac{n^2}{(n+1)^2} = |x-2| < 1$$

implies (absolute) convergence. That is, the interval of convergence is

$$|x-2|<1 \iff -1<x-2<1 \iff 1<x<3,$$

ignoring endpoints. Thus the radius of convergence is 1.

2. We now consider the two endpts:

$\underline{x=1}$: $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ converges absolutely (see $x=3$ below), and

$\underline{x=3}$: $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges absolutely because it is a p-series with $p=2$.

3. By the Ratio test,

$$\lim_{n\to\infty}\frac{\frac{|x^{n+1}|}{(n+1)!}}{\frac{|x^n|}{n!}} = \lim_{n\to\infty}\frac{|x|^{n+1}\cdot n!}{(n+1)!\cdot|x|^n} = |x|\lim_{n\to\infty}\frac{1}{n+1} = 0,$$

which is less than one independent of the value of x. Thus convergence happens for all x, that is, the interval of convergence is $(-\infty,\infty)=\mathbb{R}$.

4. (*I was expecting everyone would do* **4** *by differentiation and* **5** *by integration, but even if you did not, I graded appropriately.*)

By differentiating,

$$-\sin x = \frac{d}{dx}\cos x = 0 - x + \frac{x^3}{3!} - \frac{x^5}{5!} + \frac{x^7}{7!} - \dots,$$

so

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

5. Here

$$\int \cos x\,dx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + C,$$

that is, you do get the series for $\sin x$ but with an additional constant of integration. But of course $\int\cos x\,dx$ and $\sin x$ *do* differ by an unknown constant of integration.