# Quiz # 6 Solutions

**1.** Any method (L'Hopital's, also mult. by $1/n^4$, etc.) was accepted:

$$\lim_{n\to\infty} \frac{1-5n^4}{n^4+8n^3} = -5.$$

**2.** $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{1}{n}\right)^{1/(\ln n)} = \frac{1}{e}$, that is, it converges. In fact,

$$\ln a_n = \frac{1}{\ln n} \ln\left(\frac{1}{n}\right) = \frac{-\ln n}{\ln n} = -1$$

so also $\lim \ln a_n = -1$, and thus $\lim a_n = \lim e^{\ln a_n} = e^{-1}$.

**3.** $1.\overline{414} = 1.414\ 414\ 414\ 414\ \cdots = \frac{1413}{999}$

***Method one***: Let $x = 1.\overline{414}$. Subtracting the lines below gives $x = 1413/999$:

$$\begin{aligned} 1000x &= 1414.\overline{414} \\ x &= \quad 1.\overline{414} \\ \hline 999x &= \ 1413 \end{aligned}$$

***Method two***:

$$1.\overline{414} = 1 + \frac{414}{1000} + \frac{414}{1000^2} + \frac{414}{1000^3} + \cdots = 1 + 414\left(\frac{1}{1000} + \frac{1}{1000^2} + \frac{1}{1000^3} + \ldots\right)$$
$$= 1 + 414\left(\frac{1/1000}{1-(1/1000)}\right) = 1 + \frac{414}{999} = \frac{1413}{999}.$$

**4.** The sequence $a_n = \frac{n!}{2^n \cdot 3^n}$ diverges because

$$\frac{n!}{2^n \cdot 3^n} = \frac{n!}{6^n} = \frac{n}{6}\,\frac{n-1}{6} \cdots \frac{2}{6}\,\frac{1}{6} \geq \frac{12}{6}\,\frac{12}{6} \cdots \frac{12}{6}\,\frac{11}{6} \cdots \frac{2}{6}\,\frac{1}{6} = 2^{n-11}\left(\frac{11!}{6^{11}}\right),$$

so

$$\lim_{n\to\infty} \frac{n!}{2^n \cdot 3^n} \geq \lim_{n\to\infty} 2^{n-11}\left(\frac{11!}{6^{11}}\right) = \infty.$$