Solutions to Quiz # 4

1. [This is **7.2** # 3 with limits added.] Integration by parts, first with $u = t^2$, $dv = \cos t\, dt$, then $u' = t$, $dv' = \sin t\, dt$ gives:

$$\int_0^\pi t^2 \cos t\, dt = t^2 \sin t\Big]_0^\pi - \int_0^\pi \sin t(2t)\, dt = 0 - 2\int_0^\pi t \sin t\, dt$$
$$= -2\left(t(-\cos t)\Big]_0^\pi - \int_0^\pi (-\cos t)\, dt\right) = -2\pi - 2\int_0^\pi \cos t\, dt = -2\pi - 2(0) = -2\pi.$$

2. [This is **7.2** # 8.] Integration by parts with $u = \sin^{-1} y$, $dv = dy$, followed by ordinary substitution with $w = 1 - y^2$ gives:

$$\int \sin^{-1} y\, dy = y \sin^{-1} y - \int \frac{y\, dy}{\sqrt{1-y^2}} = y \sin^{-1} y + \frac{1}{2}\int \frac{dw}{\sqrt{w}}$$
$$= y \sin^{-1} y + \frac{1}{2} \cdot 2w^{1/2} + C = y \sin^{-1} y + \sqrt{1-y^2} + C.$$

3. (a) The correct choice is the third:

$$\frac{6x+7}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}.$$

(b) The first choice:

$$\frac{3t^2+t+4}{t^3+t} = \frac{At+B}{t^2+1} + \frac{C}{t}.$$

4. [This is **7.3** # 21 with no limits.] First I do the partial fraction decomposition:

$$\frac{1}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4} \quad \iff \quad 0\,x^2 + 0\,x + 1 = (A+B)x^2 + (B+C)x + (4A+C)$$

Equating coefficients gives $A + B = 0$, $B + C = 0$, $4A + C = 1$, which gives $A = 1/5$, $B = -1/5$, and $C = 1/5$.

Thus we can integrate:

$$\int \frac{dx}{(x+1)(x^2+4)} = \int \frac{1/5}{x+1} + \frac{(-1/5)x + (1/5)}{x^2+4}\, dx$$
$$= \frac{1}{5}\ln|x+1| - \frac{1}{5}\int \frac{x\, dx}{x^2+4} + \frac{1}{5}\int \frac{dx}{x^2+4}$$
$$= \frac{1}{5}\ln|x+1| - \frac{1}{5}\cdot\frac{1}{2}\ln(x^2+4) + \frac{1}{5}\cdot\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$
$$= \frac{1}{5}\ln|x+1| - \frac{1}{10}\ln(x^2+4) + \frac{1}{10}\tan^{-1}\left(\frac{x}{2}\right) + C$$