Solutions to Quiz # 3

1. [This is **6.4** # 8.] The separated version is $e^y dy = 3x^2 dx$ so integrating gives $e^y = x^3 + C$, that is,

$$y = \ln\left(x^3 + C\right).$$

[Note y is not equal to $\ln(x^3) + C$.]

2. [This is **6.7** # 15.] Using $(\tanh x)' = \operatorname{sech}^2 x$ and the product rule,

$$\frac{dy}{dt} = 2\,\frac{1}{2}\,t^{-1/2} \cdot \tanh(\sqrt{t}) + 2\sqrt{t} \cdot \operatorname{sech}^2(\sqrt{t})\,\frac{1}{2}\,t^{-1/2} = \frac{\tanh(\sqrt{t})}{\sqrt{t}} + \operatorname{sech}^2(\sqrt{t}).$$

3. [This is **6.7** # 51.] Using the substitution $u = \sinh x$,

$$\begin{aligned}
\int_{\ln 2}^{\ln 4} \coth x\, dx &= \int_{\ln 2}^{\ln 4} \frac{\cosh x}{\sinh x}\, dx \\
&= \int_{\sinh(\ln 2)}^{\sinh(\ln 4)} \frac{du}{u} = \int_{3/4}^{15/8} \frac{du}{u} \\
&= \ln|u|\Big]_{3/4}^{15/8} = \ln(15) - \ln 8 - \ln 3 + \ln 4 \\
&= \ln 5 + \ln 3 - 3\ln 2 - \ln 3 + 2\ln 2 \\
&= \ln 5 - \ln 2.
\end{aligned}$$

Note that

$$\sinh(\ln a) = \frac{e^{\ln a} - e^{-\ln a}}{2} = \frac{a - \frac{1}{a}}{2} = \frac{a^2 - 1}{2a},$$

which explains where "15/8" and "3/4" came from.

4. [This is **7.1** # 41.] First, $x^2 + 2x = (x+1)^2 - 1$. Thus, using $u = x + 1$,

$$\begin{aligned}
\int \frac{dx}{(x+1)\sqrt{x^2 + 2x}} &= \int \frac{dx}{(x+1)\sqrt{(x+1)^2 - 1}} \\
&= \int \frac{dx}{u\sqrt{u^2 - 1}} \\
&= \sec^{-1} u + c = \sec^{-1}(x+1) + c.
\end{aligned}$$