Solutions to Quiz # 2

1. [This is **6.3** # 41.] Start with the indefinite integral:

$$y(x) = \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + c.$$

Then find c: $0 = y(0) = \sin^{-1}(0) + c = 0 + c$ so $c = 0$. Thus $\boxed{y(x) = \sin^{-1} x}$.

2. [This is **6.3** # 7.] $\frac{dy}{dx} = \frac{1}{\tan^{-1} x}\left(\frac{1}{1+x^2}\right) = \left((1+x^2)\tan^{-1} x\right)^{-1}$.

3. $\int_0^1 \frac{4}{1+x^2}\,dx = 4\tan^{-1} x\Big|_0^1 = 4\left(\tan^{-1}(1) - \tan^{-1}(0)\right) \stackrel{\star}{=} 4\left(\frac{\pi}{4} - 0\right) = \pi$.
[One point extra credit was given for the $\star$-ed step. This integral gives a method for estimating π.]

4. (a) $\frac{dy}{dt} = (.04)y$.
(b) $y(5) = A_0 e^{(.04)5} = A_0 e^{.2}$. [And $e^{.2} \approx 1.2214$. At this low interest rate you get only 2.14% more using continuous compounded than if the interest was not compounded at all. That is, with no compounding the 4%/yr rate would be multiplied by 5 years to give 20% per 5 years and $A_0 \cdot 1.20$ dollars. Note this problem is essentially **6.4** #23; not assigned but very similar to assigned problems.]