Solutions to Quiz # 1

1. First take logarithms, then differentiate remembering product and chain rules, then rewrite as function of x on the right:

$$\begin{aligned}\ln y &= x\ln(\cos x)\\ \frac{1}{y}\frac{dy}{dx} &= 1\cdot\ln(\cos x) + x\cdot\frac{-\sin x}{\cos x}\\ \frac{dy}{dx} &= y\left(\ln(\cos x) - x\tan x\right) = (\cos x)^x\left(\ln(\cos x) - x\tan x\right).\end{aligned}$$

[Note **6.2** # 47, i.e. $y = (\sin x)^x$, was assigned and done in class.]

2. Use the substitution $u = \cos 3x$ to get a logarithmic integral:

$$\int_0^{\pi/12} 6\tan 3x\,dx = 6\int_0^{\pi/12}\frac{\sin 3x}{\cos 3x}\,dx = 6\int_1^{1/\sqrt{2}}\frac{(-1/3)\,du}{u} = -2\ln u\Big|_{u=1}^{u=1/\sqrt{2}} = \ln 2,$$

[This is **6.1** # 40. See page 462 for the technique. Note $\frac{d}{dx}\tan x = \sec^2 x$ is true but does not help with the antiderivative.]

3. Let $u = e^r + 1$:

$$\int\frac{e^r}{e^r+1}\,dr = \int\frac{du}{u} = \ln|u| + c = \ln|e^r+1| + c.$$

[This is **6.2** #31 and was assigned. Note the final answer does not simplify further because, in particular, $\ln(a+b) \neq \ln a + \ln b$.]

4. This is a case where the method of shells (section **5.2**) works more easily than discs/washers (section **5.1**). One must draw the region first (for one's own understanding). Then it helps to indicate what kind of "infinitesimal strip" you are rotating around the y-axis (vertical strip of width dx for shells and horizontal strip of height dy for washers). Rotating these strips gives a "shell" or a "washer".

By *shells*, $dV = 2\pi r\cdot h\cdot dx = 2\pi x\frac{1}{x^2}\,dx$ in this case, so

$$V = 2\pi\int_{1/2}^{2}\frac{1}{x}\,dx.$$

By *washers* we must divide into the parts below and above $y = 1/4$. Here $dV = (\pi R^2 - \pi r^2)\,dy$, so

$$V = \int_0^{1/4}\pi 2^2 - \pi(1/2)^2\,dy + \int_{1/4}^{4}\pi\left(\frac{1}{\sqrt{y}}\right)^2 - \pi(1/2)^2\,dy.$$

[This is **6.1** #73 and was assigned. The final volume is $4\pi\ln 2$, but you need not have calculated that.]