Solutions to Bonus Quiz

1. First, the absolute series $\sum_{n=1}^{\infty} \frac{n^2}{n^3+2}$ diverges by Limit Comparison to the harmonic series:

$$\lim_{n\to\infty} \frac{\frac{n^2}{n^3+2}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^3}{n^3+2} = 1.$$

But the alternating series $\sum_{n=1}^{\infty} \frac{n^2(-1)^n}{n^3+2}$ converges by the Alternating Series test because $u_n \frac{n^2}{n^3+2}$ is positive, decreasing, and has limit zero: $\lim_{n\to\infty} \frac{n^2}{n^3+2} = 0$. Thus the alternating series *converges conditionally.*

2. The series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ *converges absolutely* because we can apply the Ratio Test successfully to the absolute series $\sum_{n=0}^{\infty} \frac{1}{n!}$:

$$\lim_{n\to\infty} \frac{\frac{1}{(n+1)!}}{\frac{1}{n!}} = \lim_{n\to\infty} \frac{n!}{(n+1)!} = \lim_{n\to\infty} \frac{n!}{(n+1)(n!)} = \lim_{n\to\infty} \frac{1}{n+1} = 0.$$

Here $R = 0$ so $|R| < 1$.