1. Recall \( S = 4\pi r^2 \). We want \( dS/dr \):

\[
\frac{dS}{dr} = 8\pi r
\]

so when \( r = 3 \) we get \( \frac{dS}{dr} = 24\pi \).

2. We know \( dV/dt = 36 \). We want \( dr/dt \). Recall

\[
V = \frac{4}{3} \pi r^3.
\]

Thus

\[
\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}
\]

which is rewritten as

\[
\frac{dr}{dt} = \frac{dV/dt}{4\pi r^2} = \frac{36}{4\pi \cdot 9} = \frac{1}{\pi \text{ sec}}
\]

when \( r = 3 \text{ cm} \).

3. (a)

\[
\frac{dy}{dx} = \frac{1 \cdot \cos x - x(-\sin x)}{\cos^2 x} = \frac{\cos x + x \sin x}{\cos^2 x}
\]

(b)

\[
f'(t) = \frac{1}{3} (1 + \tan t)^{-2/3} \sec^2 t = \frac{\sec^2 t}{3 (1 + \tan t)^{2/3}}
\]

(c)

\[
g'(x) = (\ln 3)3^{(x^2)} \cdot 2x = (2 \ln 3) x 3^{(x^2)}
\]

4. (a)

\[
3x^2 + 2x y + x^2 \frac{dy}{dx} + 8y \frac{dy}{dx} = 0
\]

so

\[
\frac{dy}{dx} = \frac{-3x^2 - 2xy}{x^2 + 8y}
\]

(b) The slope is

\[
m = \left. \frac{dy}{dx} \right|_{(1,1)} = \frac{-3 - 2}{1 + 8} = -\frac{5}{9}
\]

and the equation of the tangent line is

\[
y - 1 = \left(-\frac{5}{9}\right)(x - 1).
\]
5. The function is differentiable for all $x$. The derivative is

$$f'(x) = \frac{1 \cdot (x^2 + 1) - x \cdot (2x)}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}.$$ 

The critical points are where $f'(x) = 0$, that is,

$$1 - x^2 = 0 \quad \text{or} \quad x = \pm 1.$$ 

The only critical point of interest is $x = 1$ because $x = -1$ is outside of $[0, 2]$. We make a table of $f$ values, and include the endpoints:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{1}{1+1} = \frac{1}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{2}{2^2+1} = \frac{2}{5}$</td>
</tr>
</tbody>
</table>

Thus the absolute maximum is at $x = 1$ with value $f(1) = 1/2$ while the absolute minimum is at $x = 0$ with value $f(0) = 0$.

6. [A sketch will be given in class.]

7. (a)

$$\frac{d}{dx} \sec x = \frac{d}{dx} \left( \frac{1}{\cos x} \right) = 0 \cdot \cos x - 1 \cdot (\sin x) = \frac{1 \cdot \sin x}{\cos x \cdot \cos x} = \sec x \tan x.$$ 

(b) Rewrite $y = \sec^{-1} x$ as $\sec y = x$ and do implicit differentiation:

$$\sec y \tan y \frac{dy}{dx} = 1.$$ 

Thus

$$\frac{dy}{dx} = \frac{1}{\sec y \tan y} = \frac{1}{\sec y \sqrt{\sec^2 y - 1}} = \frac{1}{x \sqrt{x^2 - 1}}$$

because $\tan^2 y + 1 = \sec^2 y$ and $\sec y = x$.

8. (a) Here $f'(x) = 1/x$ so

$$L(x) = f(a) + f'(a) (x - a) = (\ln 1) + \frac{1}{1} (x - 1) = 0 + (x - 1) = x - 1.$$ 

(b) [A sketch will be given in class.]

(c) Extra Credit We have

$$\ln 4 = f(4) \approx L(4) = 4 - 1 = 3.$$ 

This is not a very good approximation because $x = 4$ is too far from $a = 1$. In fact, $1 = \ln e^1 < \ln 4 < \ln e^2 = 2$ because $e < 4 < e^2$. So $1 < \text{error} = |3 - \ln 4| < 2$.

9. The **Mean Value Theorem**. In fact Rolle’s Theorem also works. See exercise #35 in section 4.2. Note that the Intermediate Value Theorem and the Extreme Value Theorem have nothing to do with derivatives, and thus nothing to do with speed.