Final Exam: Recent Part

100 points total for this part. You have 120 minutes for both parts together.

1. (10 pts) Find the two real numbers whose difference is 18 and whose product is a minimum.

2. (10 pts) Find \( f(x) \) for which \( f''(x) = 3x^2 - 2 \) and \( f(2) = 0 \) and \( f'(2) = 1 \).

3. (a) (8 pts) Write the integral as the limit of a sum, using right end points for the locations where the function (integrand) is evaluated:
   \[
   \int_{0}^{2} 3x - 4 \, dx
   \]
   In particular, state formulas for \( \Delta x \) and \( x_i \).

   (b) (7 pts) Compute the limit in part (a), using the fact that \( \sum_{i=1}^{n} i = \frac{1}{2} n(n + 1) \).

   (c) (7 pts) To check your work in the above parts, compute using the Fundamental Theorem of Calculus (i.e. FTC II):
   \[
   \int_{0}^{2} 3x - 4 \, dx
   \]

4. (10 pts) Compute
   \[
   \int x^2 \sec^2(x^3 + 1) \, dx
   \]

5. (10 pts) Compute \( f'(x) \) if
   \[
   f(x) = \int_{x^2}^{3} \cos(e^t) \, dt
   \]
6. Consider the curve 
\[ y = \frac{x^2 - 4}{x^2 - 1}. \]

(a) (5 pts) Is this curve even, odd, or neither?

(b) (5 pts) Find all vertical and horizontal asymptotes of the curve.

(c) (5 pts) Find the critical numbers.

(d) (5 pts) Sketch the graph.

7. (8 pts) Compute the limit:
\[ \lim_{x \to 0} \frac{\tan x - x}{1 - \cos x}. \]

8. (10 pts) Find the volume of the solid obtained by rotating the region bounded by the curves 
\[ y = x(1 - 1), \quad y = 0 \]
about the x-axis. Sketch the region, the solid, and a typical disc or washer.

Extra Credit. (3 pts) Compute 
\[ \int \frac{x^3}{\sqrt{1 - x^2}} \, dx. \]