Midterm Exam # 1: SOLUTIONS

1. (a)

(b) Formula: $(g \circ f)(x) = g(f(x)) = \sqrt{2^x - 1}$. Domain: $\{2^x - 1 \geq 0\}$, which is $\{x \geq 0\} = [0, \infty)$.

2. $F'(r) = 3r^2 + e^r$

3. (a)

$$\lim_{x \to -4} \frac{\frac{1}{4} + \frac{1}{x}}{4 + x} = \lim_{x \to -4} \frac{x + 4}{4x(4 + x)} = \lim_{x \to -4} \frac{1}{4x} = -\frac{1}{16}$$

(b)

$$\lim_{x \to \infty} \frac{x + 2}{\sqrt{9x^2 + 1}} = \lim_{x \to \infty} \frac{x + 2}{\sqrt{9x^2 + 1}} \frac{\frac{1}{x}}{\frac{1}{\sqrt{x^2}}} = \lim_{x \to \infty} \frac{1 + \frac{2}{x}}{\sqrt{9 + \frac{1}{x^2}}} = \frac{1 + 0}{\sqrt{9 + 0}} = \frac{1}{3}$$

4.

5.

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{((x+h)^2 + 2) - (x^2 + 2)}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 2 - x^2 - 2}{h} \\ &= \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} 2x + h = 2x + 0 = 2x \end{aligned}$$

6. The denominator of this fraction is $x^2 - x = x(x - 1)$, so it has zeros at $x = 0$ and at $x = 1$. These numbers are *not* zeros of the numerator. Therefore $x = 0$, $x = 1$ are both vertical lines which are vertical asymptotes of the graph. For the horizontal asymptote we compute

$$\lim_{x\to\pm\infty} \frac{x^3+8}{x^2-x} = \pm\infty.$$

Thus there is *no* horizontal asymptote.

7. (a)

$$v_{aver} = \frac{y(2)-y(1)}{2-1} = \frac{(20-1.86(2)^2)-(10-1.86(1)^2)}{1} = 10-1.86(3) = 4.42\,\frac{\text{m}}{\text{s}}$$

(b)

$$v_{inst} = \lim_{t\to 1} \frac{(10t-1.86t^2)-(10-1.86(1)^2)}{t-1} = \lim_{t\to 1} \frac{10(t-1)-1.86(t^2-1)}{t-1}$$
$$= \lim_{t\to 1} 10-1.86(t+1) = 10-1.86(2) = 6.28\,\frac{\text{m}}{\text{s}}.$$

(c)

$$\frac{dy}{dt} = 10-2(1.86)t = 10-3.72t.$$

(And note that $(dy/dt)|_{t=1} = 6.28$.)

8. The statement means that the values of $f(x)$ can be made as close as desired to L by choosing x sufficiently close to, but not equal to, a.

9. (a) $f(x) = \cos 5x$ is continuous at $x = 0$ because $x = 0$ is in the domain of f and $\lim_{x\to 0} \cos 5x$ exists and

$$\lim_{x\to 0} \cos 5x = 1 = \cos(5\cdot 0)$$

(b)

$$\lim_{x\to 0} \left(x^3 + \frac{\cos 5x}{10000}\right) = 0^3 + \frac{1}{10000} = 0.0001$$

because the function in parentheses is continuous.

10. Input $x = 10/7$ produces output $y = 7/10 = 0.7$ and input $x = 10/3$ produces output $y = 3/10 = 0.3$. The distance from $10/7$ to 2 is $2-(10/7) = 4/7$ while the distance from $10/3$ to 2 is $(10/3)-2 = 4/3$. And

$$\frac{4}{3} > \frac{4}{7},$$

a fact suggested by the picture as well. Therefore you can choose $\delta = 4/7$; you don't need to know that $4/7 = 0.57142857\ldots$. In other words, the picture shows

$$\text{if} \qquad |x-2| < \frac{4}{7} \qquad \text{then} \qquad \left|\frac{1}{x} - 0.5\right| < 0.2.$$

Extra credit. The polynomial $p(x) = x^4 + x - 3$ is continuous on the whole real line so we can use the intermediate value theorem (IVT) on any interval. I plugged in some values, starting with $x = 0$. Note $p(0) = -3$ is negative, but the function goes to $+\infty$ as $x \to \pm\infty$. Also I notice that $p(-2) = +9$ and $p(2) = +15$. Therefore, using the IVT on the interval $[-2, 0]$ with $L = 0$ we conclude there is a solution between -2 and 0. By the same argument there is another solution on the interval $[0, 2]$.